

# Characteristic Basis Function Patterns Method for Reflector Antenna Calibration: An Extension to Multiple Frequencies

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## Abstract

A simple and effective method to extend the characteristic basis function pattern (CBFP) method to multiple frequencies is presented. Transformation matrices are generated from simulated basis functions at each frequency of interest. By computing model coefficients, by means of a singular value decomposition (SVD) to enhance accuracy over a wide bandwidth, using a few directional measurements of any given pattern at a single frequency, linear algebra is applied to extract expansion coefficients at any other frequency within the model's range by utilising the transformation matrices. For cases where a large number of basis functions is required,  $k$  sparse approximation of the expansion coefficients is used to improve accuracy. The full radiation pattern at all frequencies is then recovered from a single frequency measurement using the extracted expansion coefficients. Simulation results show a maximum pattern recovery error below 40 dB across a bandwidth ratio of up to 10 : 1.

**Index Terms:** calibration, characteristic basis function patterns, radio astronomy.



## Motivation and Objective

- ▶ Third generation calibration algorithms require knowledge of radiation patterns over a wide angular region.
- ▶ CBFP model provides accurate pattern models and require very few measurements.
- ▶ CBFP model has to be extracted at **each frequency**, requiring repeated measurements for a wideband system.
- ▶ We present an efficient extension of the CBFP method to multiple frequencies **without the need for repeated measurements**. Only a few points at a **single frequency** need to be measured.

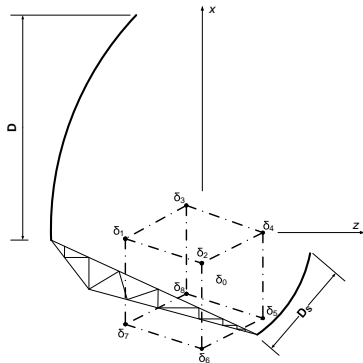


## CBFP Method

Pattern approximated as a linear combination of a few basis functions as<sup>1</sup>

$$F_r(\Omega, s) = \sum_{n=1}^N \alpha_n f_n(\Omega, s), \quad (1)$$

- ▶  $F_r(\Omega)$  – Reconstructed pattern at frequency  $s$ .
- ▶  $\alpha_n$  – Model coefficients.
- ▶  $f_n$  – Basis functions given by EM simulations of deformed system. i.e., moving feed/sub-reflector to positions  $\delta_i$ .
- ▶ The idea is that any pattern resulting from system deformation within the model's range can be recovered using (1).



<sup>1</sup>Maaskant *et al.*, IEEE TAP 2012; Young *et al.*, IEEE TAP 2013

## CBFP Extension<sup>2</sup>

From (1), We can expand model coefficients as

$$\begin{bmatrix} \alpha_1 \\ \vdots \\ \alpha_N \end{bmatrix} = \underbrace{\begin{bmatrix} \tau_{11} & \cdots & \tau_{1N} \\ \vdots & \ddots & \vdots \\ \tau_{N1} & \cdots & \tau_{NN} \end{bmatrix}}_W \begin{bmatrix} \lambda_1 \\ \vdots \\ \lambda_N \end{bmatrix}$$

1. For a linear system, it suffices to determine  $\lambda = [\lambda_1, \dots, \lambda_N]$ , at one frequency  $s_j$ , in order to recover model coefficients at an arbitrary frequency  $s_p$  if  $W$  is known at  $s_p$ .
2. The objective of this work is to determine a suitable  $W$  matrix at multiple frequencies.

<sup>2</sup>N. Mutonkole and D.I.L. de Villiers, EuCAP 2015.

### Algorithm

1. Derive a set of interpolation matrices  $W(s_j)$  at multiple frequencies from simulated CBFPs

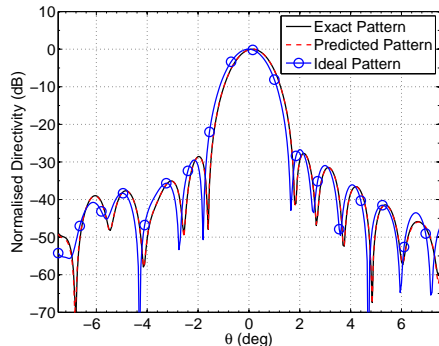
$$T = \{W(s_1), W(s_2), \dots, W(s_Q)\},$$

2. From measurements at one frequency  $s_p$  and use the set  $T$  to approximate model coefficients at all other frequency points  $s_{\{1, \dots, Q\}}$ .
3. Dimensionality reduction via  $k$ -sparse approximation.



## Support Arm Deformation - Example I: Gaussian Feed

- ▶ Model pattern drifts due to support arm deformation on an offset Gregorian antenna.
- ▶ Recovered pattern at  $f = 1.217$  GHz obtained by taking measurements at 0.736 GHz.
- ▶ Max. Error =  $-41.98$  dB.



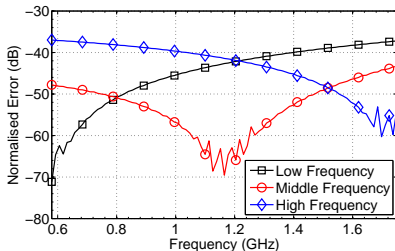
Predicted pattern in the  $\phi = 150^\circ$  plane. The ideal pattern corresponds to the primary basis function (ideal location of feed and sub-reflector) at 1.217 GHz.



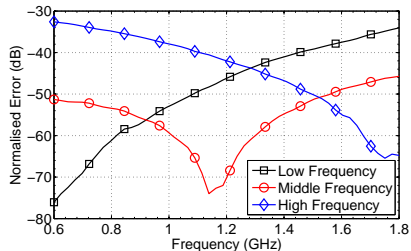
## Support Arm Deformation - Example I: Where to measure?

Measuring at or near centre frequency leads to least pattern recovery error (below  $-43$  dB) over 3 : 1 bandwidth, with ideal or real feed<sup>3</sup>.

► Ideal Gaussian feed.



► Frequency-dependent Sinuous feed.



<sup>3</sup>N. Mutonkole and D.I.L. de Villiers, AFRICON 2013.

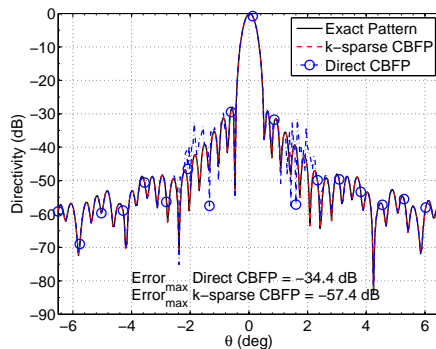




## Decade Bandwidth

- ▶ Model positional errors of the feed in a 2D plane.
- ▶ Large number of basis functions required to "linearize" system due to large bandwidth.
- ▶ Dimensionality problem addressed by enforcing sparsity in the coefficients vector ( $k$ -sparse approximation).
- ▶ Coefficients with low magnitude (below a certain threshold) contribute to noise in the pattern reconstruction process.

- ▶ Clear improvement with  $k$ -sparse approx.

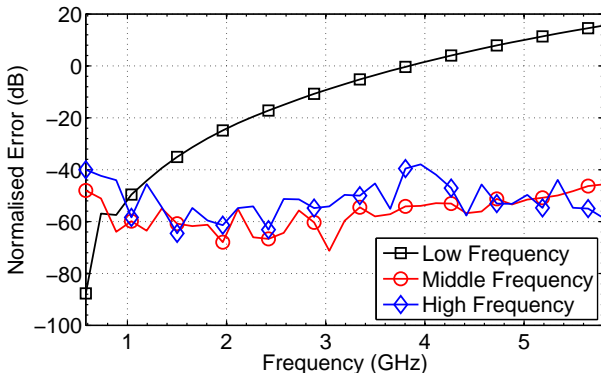


At  $f = 4.11$  GHz.



## Decade Bandwidth - Errors

As with previous cases, sampling at/near the centre frequency leads to smallest modelling errors ( $< -45$  dB) across the band.



# References

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