Forum for Electromagnetic Research Methods and Application Technologies (FERMAT)

Analysis of Scattering from Ferromagnetic Objects Using Landau-Lifshitz-Gilbert and Volume Integral Equations.

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Abstract: An explicit marching on-in-time scheme for analyzing transient electromagnetic wave interactions on ferromagnetic scatterers is described. The proposed method solves a coupled system of time domain magnetic field volume integral and Landau-Lifshitz-Gilbert (LLG) equations. The unknown fluxes and fields are discretized using full and half Schaubert-Wilton-Glisson functions in space and bandlimited temporal interpolation functions in time. The coupled system is cast in the form of an ordinary differential equation and integrated in time using a PE(CE)^m type linear multistep method to obtain the unknown expansion coefficients. Numerical results demonstrating the stability and accuracy of the proposed scheme are presented.

Keywords: Landau-Lifshitz-Gilbert equations, Transient analysis, Integral equations.

Sadeed Bin Sayed received the B.Tech. degree in Electronics and Communication Engineering from the National Institute of Technology Calicut, India, in August 2004 and the M.Tech. degree in Communications Engineering from Indian Institute of Technology Delhi, India in August 2007. He worked from July 2007 to May 2008, as a member of technical staff and from May 2008 to February 2010, as an engineer in Digibee Microsystems and in Qualcomm India Pvt. Ltd respectively. Currently, he is pursuing the Ph.D. degree at the Division of Physical Sciences and Engineering at the King Abdullah University of Science and Technology (KAUST), Saudi Arabia. He won Honorable Mention Award twice at Symposium on Antennas and Propagation (APS), July 2014 and July 2015 and was one of the student paper finalists in June 2016. He ranked 3rd in the student paper at the International Conference on Review of Progress in Applied Computational Electromagnetics (ACES), March 2014. His research interests are in the field of time-domain volume integral equations and their marching on-in-time based solutions.

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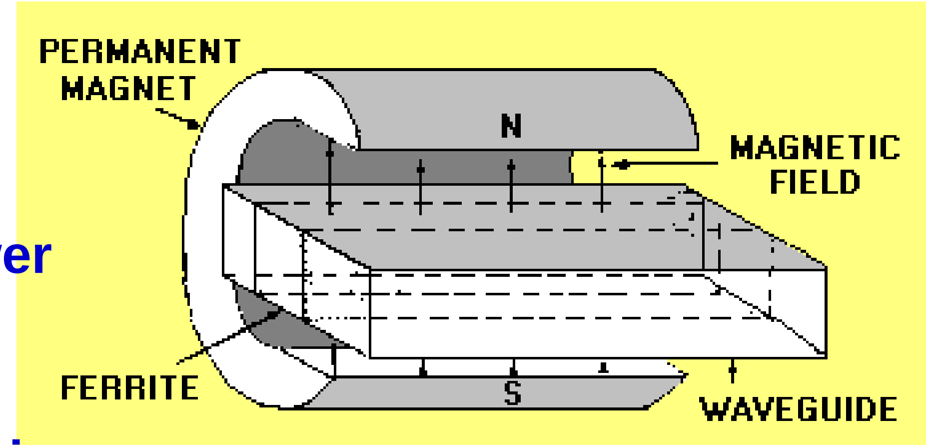
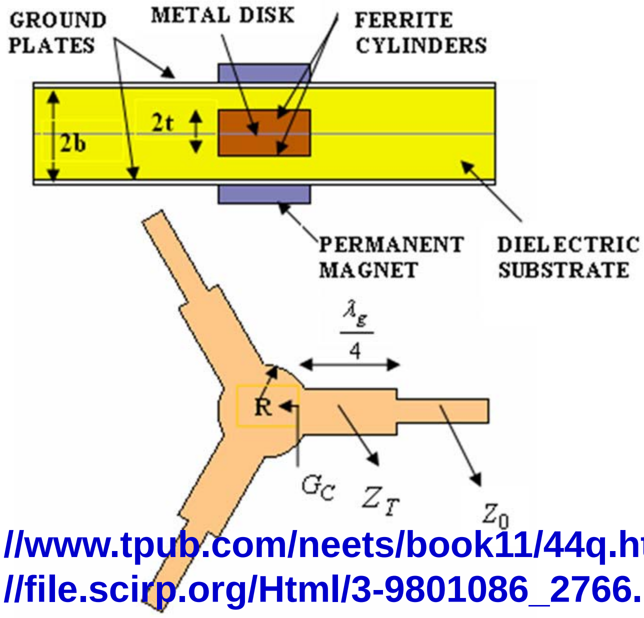
Outline

- Introduction
  - Applications
  - Existing methods
  - Motivation
- Formulation
  - Coupled LLG and VIE
  - Discretization
  - Coupled matrix system
- Numerical Results
  - Verification and convergence
- Conclusion

Applications

- **Isolator**[1]
  - Microwave signal can only be transmitted in one direction
  - Protection against reverse power flow
- **Circulator**[2]
  - Waves can only be transmitted in rotation
  - Switch in between transmitter and receiver
- **Phase shifter**
  - DC magnetic field to change the phase of the wave
- **Phased arrays**
  - Reconfigurable Antennas
  - DC magnetic field for dynamic tuning





[1] http://www.tpub.com/neets/book11/44q.htm
[2] http://file.scirp.org/Html/3-9801086_2766.htm

Existing Methods

- Full wave simulation tools are used to design and characterize these systems
- LLG should be considered while solving Maxwell equations for fields/currents.
- Differential equation methods
  - LLG is converted into a tensor and introduced as a constitutive relation between magnetic flux and field
  - LLG is introduced as an auxiliary equation
- Integral equation methods
  - LLG is converted into a tensor and introduced as a constitutive relation between magnetic flux and field

Existing Methods

- **Tensor method**
  - Reduced number of unknowns
  - Simplified via small signal approximation
  - Accurate only around saturation point
  - Dispersive
  - Difficult and costly to implement in time domain

- **Auxiliary equation method**
  - Flexible: Non-linearity can be modeled
  - Higher number of unknowns

Motivation

- **Differential equation methods**
  - Volumetric discretization of the whole computation domain
  - Approximate absorbing boundary conditions (PML, ABC)
  - Time step size should satisfy Courant condition

- **Integral Equation methods**
  - Discretization of scatterer
  - Implicitly satisfy radiation condition
  - Time step size should resolve only the incident field’s maximum frequency
  - LLG is introduced as an auxiliary equation

Formulation

- **Scattering problem**

$\mathbf{M}_{eq}(\mathbf{r},t)$: Equivalent current density

$$\mathbf{M}_{eq}(\mathbf{r},t) = \partial_t \mathbf{B}(\mathbf{r},t) - \mu_0 \partial_t \mathbf{H}(\mathbf{r},t)$$

$\mathbf{H}^{\text{inc}}(\mathbf{r},t)$: Incident magnetic field intensity

$\mathbf{H}^{\text{sca}}(\mathbf{r},t)$: Scattered magnetic field intensity

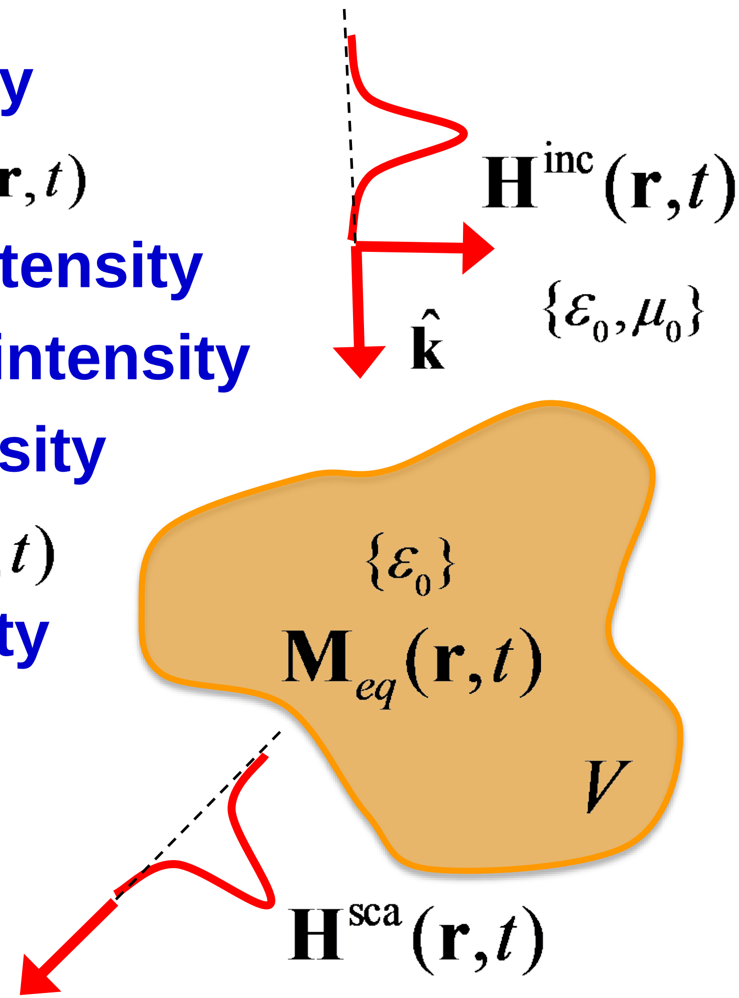
$\mathbf{H}(\mathbf{r},t)$: Total magnetic field intensity

$$\mathbf{H}(\mathbf{r},t) = \mathbf{H}^{\text{inc}}(\mathbf{r},t) + \mathbf{H}^{\text{sca}}(\mathbf{r},t)$$

$\mathbf{B}(\mathbf{r},t)$: Total magnetic flux density

ε_0 : Free space permittivity

μ_0 : Free space permeability



Formulation

- **TD-MFVIE**

$$\partial_t \mathbf{H}^{\text{inc}}(\mathbf{r},t) = \partial_t \mathbf{H}(\mathbf{r},t) + \mathcal{L}\{\mathbf{H}(\mathbf{r},t)\} - \mathcal{L}\{\mu_0^{-1}\mathbf{B}(\mathbf{r},t)\}$$

$$\mathbf{r} \in V$$

$$\mathcal{L}\{\mathbf{X}(\mathbf{r},t)\} = -\frac{\varepsilon_0\mu_0}{4\pi}\int_V \frac{1}{R}\partial_{t'}^3 \mathbf{X}(\mathbf{r}',t')\Big|_{t'=t-R/c_0} d\mathbf{r}' + \frac{1}{4\pi}\nabla\int_V \frac{1}{R}[\nabla'\cdot\partial_{t'}\mathbf{X}(\mathbf{r}',t')]_{t'=t-R/c_0} d\mathbf{r}'$$

$$R = |\mathbf{r}-\mathbf{r}'|$$

$$c_0 = 1\big/\sqrt{\varepsilon_0\mu_0}$$

- **LLG**

$$\partial_t \mathbf{M}_t(\mathbf{r},t) = -\gamma \mathbf{M}_t(\mathbf{r},t)\times \mathbf{H}_t(\mathbf{r},t) + \frac{\gamma\alpha}{|\mathbf{M}_t(\mathbf{r},t)|}\mathbf{M}_t(\mathbf{r},t)\times\{\mathbf{M}_t(\mathbf{r},t)\times\mathbf{H}_t(\mathbf{r},t)\} \qquad \mathbf{r}\in V$$

α : Gyro-magnetic ratio
γ : Damping constant

Discretization

- **Expansion of unknowns**

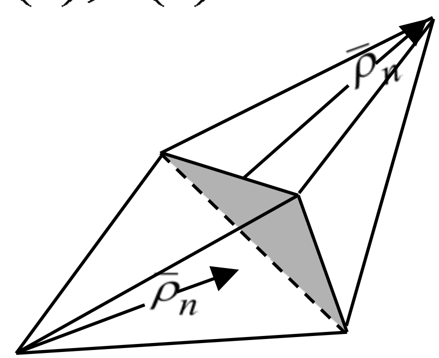
$$\mathbf{H}(\mathbf{r},t)=\sum_{n=1}^{N}\{\mathbf{L}(t)\}_n\mathbf{f}_n^{\mathrm{h}}(\mathbf{r})$$

$$\mathbf{B}(\mathbf{r},t)=\sum_{n=1}^{N}\{\mathbf{I}(t)\}_n\mathbf{f}_n^{\mathrm{f}}(\mathbf{r})$$

Half and full SWG functions : $\mathbf{f}_n^{\mathrm{h}}(\mathbf{r}),\mathbf{f}_n^{\mathrm{f}}(\mathbf{r})$

Time dependent unknown coefficients : $\mathbf{L}(t),\mathbf{I}(t)$

$$\mathbf{f}_n^{\mathrm{f/h}}(\mathbf{r})=\frac{A_k}{3V_k^{\pm}}(\mathbf{r}-\mathbf{r}_k^{\pm}),\ \mathbf{r}\in V_k^{\pm}$$



[3]D. H. Schaubert, *et al.*, *IEEE Trans. Antennas Propag.*, 32(1), pp. 77–85, 1984.

Discretization

- Spatially testing with $\mathbf{f}_m^{\mathrm{h}}(\mathbf{r})$

$$\mathbf{G}^{\mathrm{hh}}\,\partial_t \mathbf{L}(t) = \mathbf{V}^{\mathrm{inc}}(t) + \mathbf{V}_{\mathrm{H}}^{\mathrm{sca}}(t, \mathbf{L}(t)) + \mathbf{V}_{\mathrm{B}}^{\mathrm{sca}}(t, \mathbf{I}(t))$$

- Gram matrix

$$\{\mathbf{G}^{\mathrm{hh}}\}_{m,n} = \int_{V_m} \mathbf{f}_m^{\mathrm{h}}(\mathbf{r}) \cdot \mathbf{f}_n^{\mathrm{h}}(\mathbf{r})\, d\mathbf{r}$$

- Tested incident field

$$\{\mathbf{V}^{\mathrm{inc}}(t)\}_m = \int_{V_m} \mathbf{f}_m^{\mathrm{h}}(\mathbf{r}) \cdot \partial_t \mathbf{H}^{\mathrm{inc}}(\mathbf{r}, t)\, d\mathbf{r}$$

Discretization

- **Spatially testing with $\mathbf{f}_m^{\mathrm{h}}(\mathbf{r})$**

$$\mathbf{G}^{\mathrm{hh}}\,\partial_t \mathbf{L}(t) = \mathbf{V}^{\mathrm{inc}}(t) + \mathbf{V}_{\mathrm{H}}^{\mathrm{sca}}(t,\mathbf{L}(t)) + \mathbf{V}_{\mathrm{B}}^{\mathrm{sca}}(t,\mathbf{I}(t))$$

- **Tested scattered field**

$$\{\mathbf{V}_{\mathrm{H}}^{\mathrm{sca}}(t,\mathbf{L}(t))\}_m = \sum_{n=1}^{N} \int_{V_m} \mathbf{f}_m^{\mathrm{h}}(\mathbf{r}) \cdot \mathcal{L}\left\{\{\mathbf{L}(t)\}_n \mathbf{f}_n^{\mathrm{h}}(\mathbf{r})\right\} d\mathbf{r}$$

$$\{\mathbf{V}_{\mathrm{B}}^{\mathrm{sca}}(t,\mathbf{I}(t))\}_m = -\sum_{n=1}^{N} \int_{V_m} \mathbf{f}_m^{\mathrm{h}}(\mathbf{r}) \cdot \mathcal{L}\left\{\mu_0^{-1}\{\mathbf{I}(t)\}_n \mathbf{f}_n^{\mathrm{f}}(\mathbf{r})\right\} d\mathbf{r}$$

Discretization

- **Temporal sampling with time step size Δt**

$$\mathbf{L}_i = \mathbf{L}(i\Delta t)$$

$$\mathbf{I}_i = \mathbf{I}(i\Delta t)$$

- **To accurately determine the retarded-time integrals in operator $\mathcal{L}\{\cdot\}$, $\mathbf{L}_i$ and $\mathbf{I}_i$ should be interpolated**

$$\mathbf{L}(t) = \sum_{i=1}^{N_t} T_i(t)\mathbf{L}_i = \sum_{i=1}^{N_t} T(t - i\Delta t)\mathbf{L}_i$$

$$\mathbf{I}(t) = \sum_{i=1}^{N_t} T_i(t)\mathbf{I}_i = \sum_{i=1}^{N_t} T(t - i\Delta t)\mathbf{I}_i$$

$T(t)$ **: Temporal interpolation functions**

Discretization

- **MOT matrix equation**

$$\mathbf{G}^{\mathrm{hh}}\,\partial_t \mathbf{L}_j = \mathbf{Z}_0^{\mathrm{hh}}\mathbf{L}_j - \mu_0^{-1}\mathbf{Z}_0^{\mathrm{hf}}\mathbf{I}_j \underbrace{+\mathbf{V}_j^{\mathrm{inc}} + \sum_{i=1}^{j-1}\mathbf{Z}_{j-i}^{\mathrm{hh}}\mathbf{L}_i - \sum_{i=1}^{j-1}\mu_0^{-1}\mathbf{Z}_{j-i}^{\mathrm{hf}}\mathbf{I}_i}_{\mathbf{V}_j^{\mathrm{fixed}}}$$

- **Matrix elements**

$$\{\mathbf{Z}_{j-i}^{\mathrm{hh}}\}_{m,n} = \int_{V_m} \mathbf{f}_m^{\mathrm{h}}(\mathbf{r}) \cdot \mathcal{L}\{\mathbf{f}_m^{\mathrm{h}}(\mathbf{r}) T_i(t)\}\Big|_{t=j\Delta t}\, d\mathbf{r}$$

$$\{\mathbf{Z}_{j-i}^{\mathrm{hf}}\}_{m,n} = \int_{V_m} \mathbf{f}_m^{\mathrm{h}}(\mathbf{r}) \cdot \mathcal{L}\{\mathbf{f}_m^{\mathrm{f}}(\mathbf{r}) T_i(t)\}\Big|_{t=j\Delta t}\, d\mathbf{r}$$

Discretization

- **LLG**

$$\partial_t \mathbf{M}_t(\mathbf{r},t) = -\gamma \mathbf{M}_t(\mathbf{r},t) \times \mathbf{H}_t(\mathbf{r},t) + \frac{\gamma\alpha}{|\mathbf{M}_t(\mathbf{r},t)|} \mathbf{M}_t(\mathbf{r},t) \times \{\mathbf{M}_t(\mathbf{r},t) \times \mathbf{H}_t(\mathbf{r},t)\}$$

α : Gyro-magnetic ratio
γ : Damping constant

$$\mathbf{M}_t(\mathbf{r},t) = \mu_0^{-1} \mathbf{B}_t(\mathbf{r},t) - \mathbf{H}_t(\mathbf{r},t)$$

- **Lossless:** $\partial_t \mathbf{M}_t(\mathbf{r},t) = -\gamma \mathbf{M}_t(\mathbf{r},t) \times \mathbf{H}_t(\mathbf{r},t)$

- **With external DC bias field**

$$\mathbf{H}_t(\mathbf{r},t) = \mathbf{H}_{\mathrm{dc}}(\mathbf{r}) + \mathbf{H}(\mathbf{r},t)$$

$$\mathbf{M}_t(\mathbf{r},t) = \mathbf{M}_{\mathrm{s}}(\mathbf{r}) + \mu_0^{-1} \mathbf{B}(\mathbf{r},t) - \mathbf{H}(\mathbf{r},t)$$

Discretization

- **Under small signal approximation**

$$\partial_t \mathbf{M}(\mathbf{r},t) = \gamma \mathbf{H}_{\mathrm{dc}}(\mathbf{r}) \times \mu_0^{-1}\mathbf{B}(\mathbf{r},t) - \gamma\left[\mathbf{M}_{\mathrm{s}}(\mathbf{r}) + \mathbf{H}_{\mathrm{dc}}(\mathbf{r})\right] \times \mathbf{H}(\mathbf{r},t)$$

- **Auxiliary equation**

$$\mu_0^{-1}\partial_t \mathbf{B}(\mathbf{r},t) - \partial_t \mathbf{H}(\mathbf{r},t) = \gamma \mathbf{H}_{dc}(\mathbf{r}) \times \mu_0^{-1}\mathbf{B}(\mathbf{r},t) - \gamma\left[\mathbf{M}_s(\mathbf{r}) + \mathbf{H}_{dc}(\mathbf{r})\right] \times \mathbf{H}(\mathbf{r},t)$$

Discretization

- **Spatially tested with $\mathbf{f}_m^{\mathrm{f}}(\mathbf{r})$ and sampled in time**

$$\mu_0^{-1}\mathbf{G}^{\mathrm{ff}}\,\partial_t\mathbf{I}_j-\mathbf{G}^{\mathrm{fh}}\,\partial_t\mathbf{L}_j=\mathbf{H}_{\mathrm{DC}}\mu_0^{-1}\mathbf{I}_j-\mathbf{M}_{\mathrm{S}}\mathbf{L}_j$$

- **Matrix elements**

$$\{\mathbf{H}_{\mathrm{DC}}\}_{m,n}=\int_{V_m}\mathbf{f}_m^{\mathrm{f}}(\mathbf{r})\cdot\gamma\mathbf{H}_{dc}(\mathbf{r})\times\mathbf{f}_n^{\mathrm{h}}(\mathbf{r})d\mathbf{r}$$

$$\{\mathbf{M}_{\mathrm{S}}\}_{m,n}=\int_{V_m}\mathbf{f}_m^{\mathrm{f}}(\mathbf{r})\cdot\gamma\left[\mathbf{M}_s(\mathbf{r})+\mathbf{H}_{dc}(\mathbf{r})\right]\times\mathbf{f}_n^{\mathrm{h}}(\mathbf{r})d\mathbf{r}$$

$$\{\mathbf{G}^{\mathrm{ff}}\}_{m,n}=\int_{V_m}\mathbf{f}_m^{\mathrm{f}}(\mathbf{r})\cdot\mathbf{f}_n^{\mathrm{f}}(\mathbf{r})d\mathbf{r}\quad\{\mathbf{G}^{\mathrm{fh}}\}_{m,n}=\int_{V_m}\mathbf{f}_m^{\mathrm{f}}(\mathbf{r})\cdot\mathbf{f}_n^{\mathrm{h}}(\mathbf{r})d\mathbf{r}$$

Discretization

- **MOT matrix equation**

$$\mathbf{G}^{\mathrm{hh}}\partial_t\mathbf{L}_j=\mathbf{Z}_0^{\mathrm{hh}}\mathbf{L}_j-\mu_0^{-1}\mathbf{Z}_0^{\mathrm{hf}}\mathbf{I}_j\underbrace{+\mathbf{V}_j^{\mathrm{inc}}+\sum_{i=1}^{j-1}\mathbf{Z}_{j-i}^{\mathrm{hh}}\mathbf{L}_i-\sum_{i=1}^{j-1}\mu_0^{-1}\mathbf{Z}_{j-i}^{\mathrm{hf}}\mathbf{I}_i}_{\mathbf{V}_j^{\mathrm{fixed}}}$$

- **Constitutive equation**

$$\mu_0^{-1}\mathbf{G}^{\mathrm{ff}}\partial_t\mathbf{I}_j-\mathbf{G}^{\mathrm{fh}}\partial_t\mathbf{L}_j=\mathbf{H}_{\mathrm{DC}}\mu_0^{-1}\mathbf{I}_j-\mathbf{M}_{\mathrm{S}}\mathbf{L}_j$$

MOT Formulation

- Coupled matrix system

$$\begin{bmatrix} \mathbf{G}^{\mathrm{hh}} & \mathbf{0} \\ -\mathbf{G}^{\mathrm{fh}} & \mu_0^{-1}\mathbf{G}^{\mathrm{ff}} \end{bmatrix} \begin{bmatrix} \partial_t \mathbf{L}_j \\ \partial_t \mathbf{I}_j \end{bmatrix} = \begin{bmatrix} \mathbf{Z}_0^{\mathrm{hh}}\mathbf{L}_j - \mu_0^{-1}\mathbf{Z}_0^{\mathrm{hf}}\mathbf{I}_j + \mathbf{V}_j^{\mathrm{fixed}} \\ \mathbf{H}_{\mathrm{DC}}\mu_0^{-1}\mathbf{I}_j - \mathbf{M}_{\mathrm{S}}\mathbf{L}_j \end{bmatrix}$$

- Relates time derivative of samples to samples
- Integrated in time using predictor-corrector methods – $\mathrm{PE(CE)}^m$

Explicit MOT

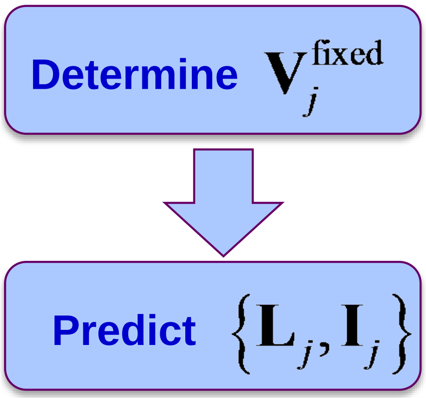
- At time step j

Determine $\mathbf{V}_j^{\text{fixed}}$

$$\mathbf{V}_j^{\text{fixed}} = \mathbf{V}_j^{\text{inc}} + \sum_{i=1}^{j-1} \mathbf{Z}_{j-i}^{\text{hh}} \mathbf{L}_i - \sum_{i=1}^{j-1} \mu_0^{-1} \mathbf{Z}_{j-i}^{\text{hf}} \mathbf{I}_i$$

Explicit MOT

- At time step j



$$\mathbf{L}_j^0 = \sum_{l=1}^{k} \left[\{\mathbf{p}\}_l \mathbf{L}_{j-1+l-k} + \{\mathbf{p}\}_{k+l} \partial_t \mathbf{L}_{j-1+l-k} \right]$$

$$\mathbf{I}_j^0 = \sum_{l=1}^{k} \left[\{\mathbf{p}\}_l \mathbf{I}_{j-1+l-k} + \{\mathbf{p}\}_{k+l} \partial_t \mathbf{I}_{j-1+l-k} \right]$$

Explicit MOT

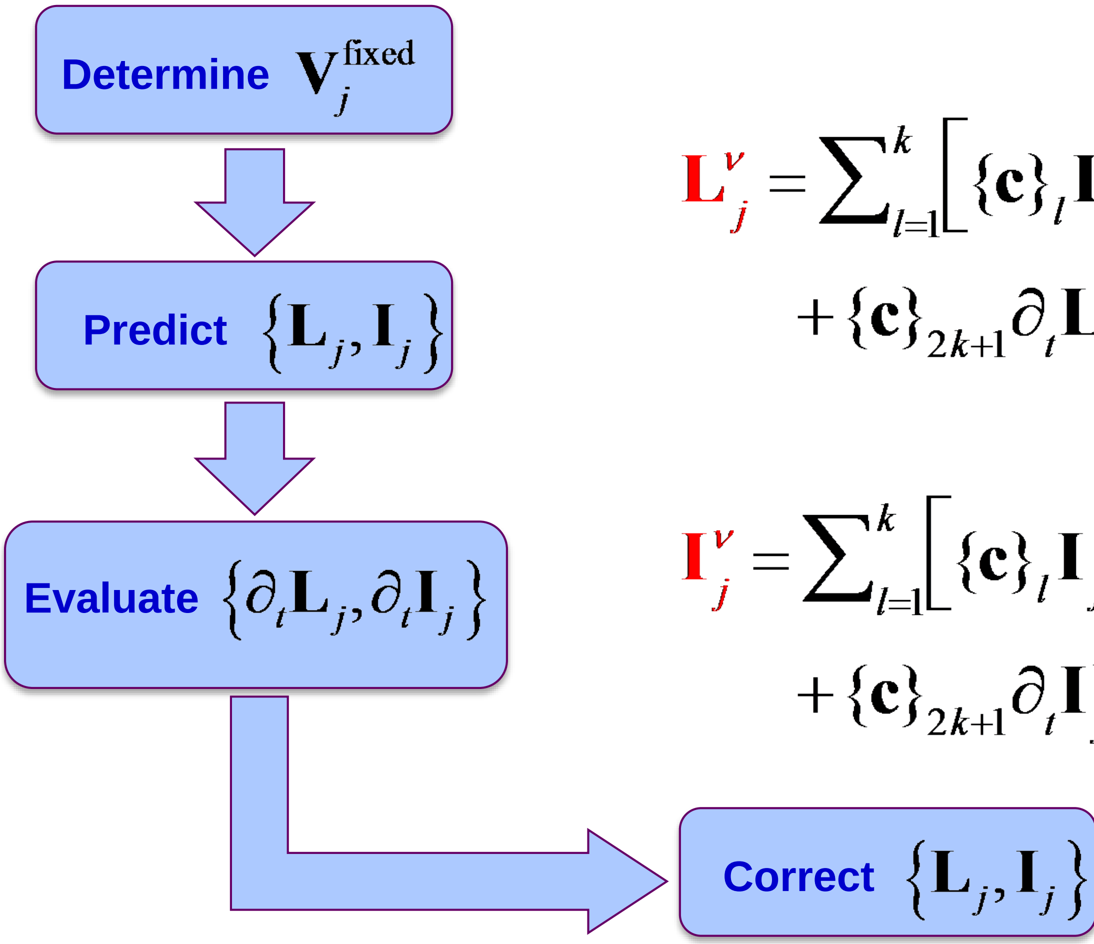
- At time step j

1. Determine $\mathbf{V}_j^{\text{fixed}}$
2. Predict $\{\mathbf{L}_j, \mathbf{I}_j\}$
3. Evaluate $\{\partial_t \mathbf{L}_j, \partial_t \mathbf{I}_j\}$

$$\begin{bmatrix} \mathbf{G}^{\text{hh}} & \mathbf{0} \\ -\mathbf{G}^{\text{fh}} & \mu_0^{-1}\mathbf{G}^{\text{ff}} \end{bmatrix} \begin{bmatrix} \partial_t \mathbf{L}_j \\ \partial_t \mathbf{I}_j \end{bmatrix} = \begin{bmatrix} \mathbf{Z}_0^{\text{hh}}\mathbf{L}_j - \mu_0^{-1}\mathbf{Z}_0^{\text{hf}}\mathbf{I}_j + \mathbf{V}_j^{\text{fixed}} \\ \mathbf{H}_{\text{DC}}\mu_0^{-1}\mathbf{I}_j - \mathbf{M}_{\text{S}}\mathbf{L}_j \end{bmatrix}$$

Explicit MOT

- At time step j



$$\mathbf{L}_j^{\nu} = \sum_{l=1}^{k} \left[\{\mathbf{c}\}_l \mathbf{L}_{j-1+l-k} + \{\mathbf{c}\}_{k+l} \partial_t \mathbf{L}_{j-1+l-k} \right] + \{\mathbf{c}\}_{2k+1} \partial_t \mathbf{L}_j^{\nu-1} \quad ; \nu \geq 1$$

$$\mathbf{I}_j^{\nu} = \sum_{l=1}^{k} \left[\{\mathbf{c}\}_l \mathbf{I}_{j-1+l-k} + \{\mathbf{c}\}_{k+l} \partial_t \mathbf{I}_{j-1+l-k} \right] + \{\mathbf{c}\}_{2k+1} \partial_t \mathbf{I}_j^{\nu-1} \quad ; \nu \geq 1$$

Explicit MOT

- At time step j

Determine $\mathbf{V}_j^{\text{fixed}}$

Predict $\{\mathbf{L}_j, \mathbf{I}_j\}$

Evaluate $\{\partial_t \mathbf{L}_j, \partial_t \mathbf{I}_j\}$

Correct $\{\mathbf{L}_j, \mathbf{I}_j\}$

Apply SOR $\{\mathbf{L}_j, \mathbf{I}_j\}$

$$\mathbf{L}_j^{\nu} = \alpha \mathbf{L}_j^{\nu} + (1-\alpha)\mathbf{L}_j^{\nu-1}$$

$$\mathbf{I}_j^{\nu} = \alpha \mathbf{I}_j^{\nu} + (1-\alpha)\mathbf{I}_j^{\nu-1}$$

Explicit MOT

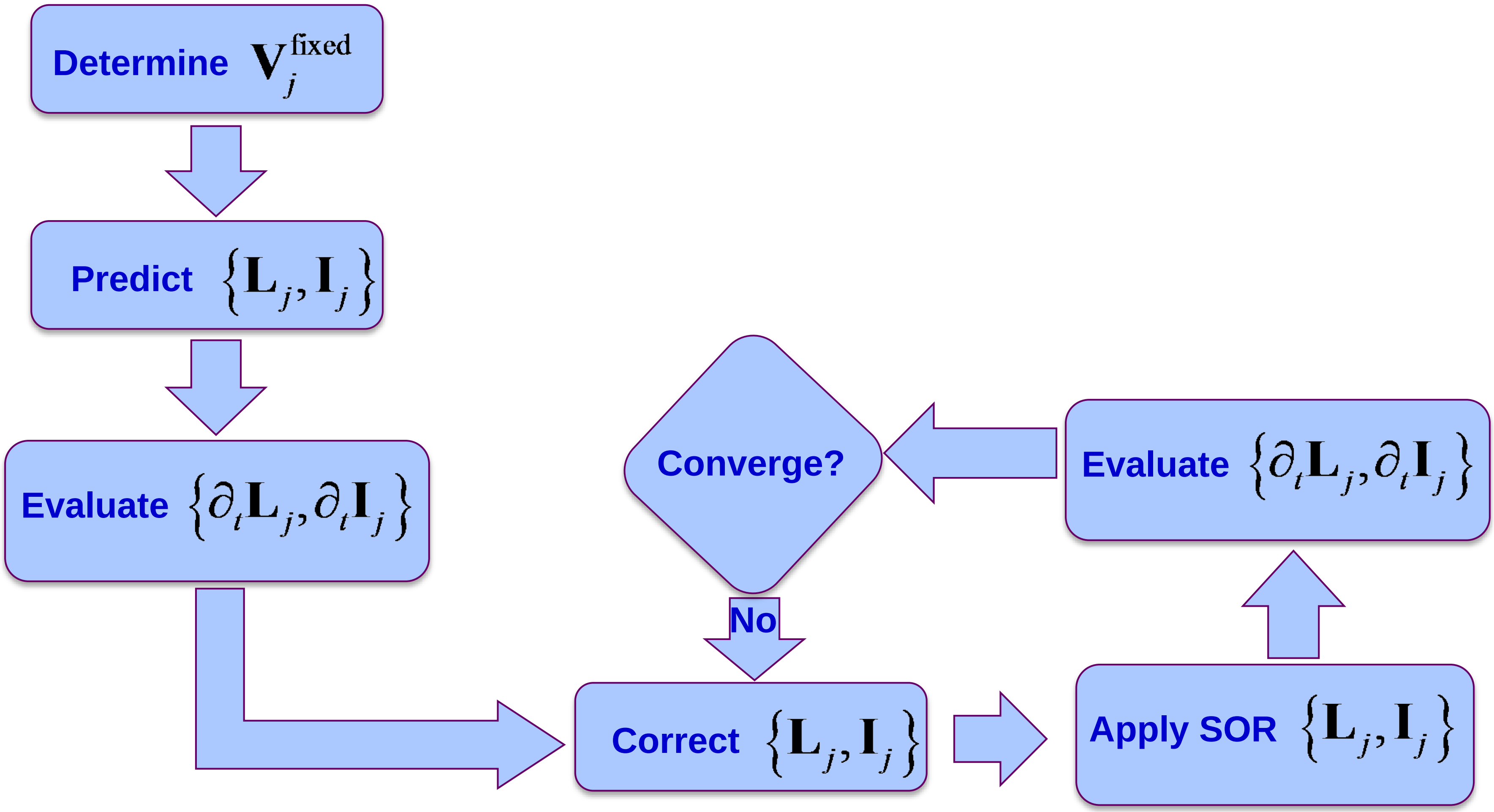
- At time step j

$$\begin{bmatrix} \mathbf{G}^{\mathrm{hh}} & \mathbf{0} \\ -\mathbf{G}^{\mathrm{fh}} & \mu_0^{-1}\mathbf{G}^{\mathrm{ff}} \end{bmatrix} \begin{bmatrix} \partial_t \mathbf{L}_j \\ \partial_t \mathbf{I}_j \end{bmatrix} = \begin{bmatrix} \mathbf{Z}_0^{\mathrm{hh}}\mathbf{L}_j - \mu_0^{-1}\mathbf{Z}_0^{\mathrm{hf}}\mathbf{I}_j + \mathbf{V}_j^{\mathrm{fixed}} \\ \mathbf{H}_{\mathrm{DC}}\mu_0^{-1}\mathbf{I}_j - \mathbf{M}_{\mathrm{S}}\mathbf{L}_j \end{bmatrix}$$

1. Determine $\mathbf{V}_j^{\mathrm{fixed}}$
2. Predict $\{\mathbf{L}_j, \mathbf{I}_j\}$
3. Evaluate $\{\partial_t \mathbf{L}_j, \partial_t \mathbf{I}_j\}$
4. Correct $\{\mathbf{L}_j, \mathbf{I}_j\}$
5. Apply SOR $\{\mathbf{L}_j, \mathbf{I}_j\}$
6. Evaluate $\{\partial_t \mathbf{L}_j, \partial_t \mathbf{I}_j\}$

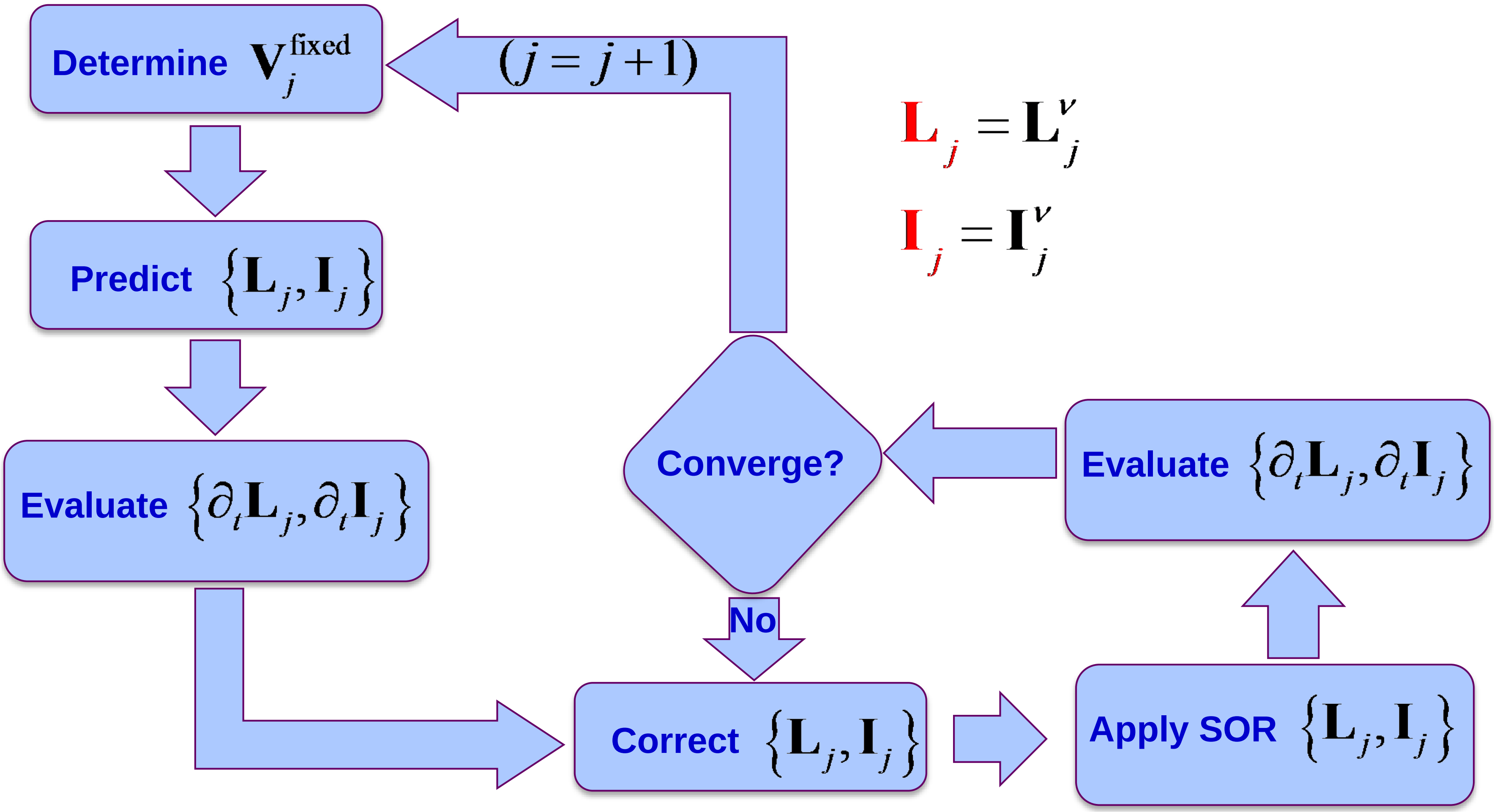
Explicit MOT

- At time step j



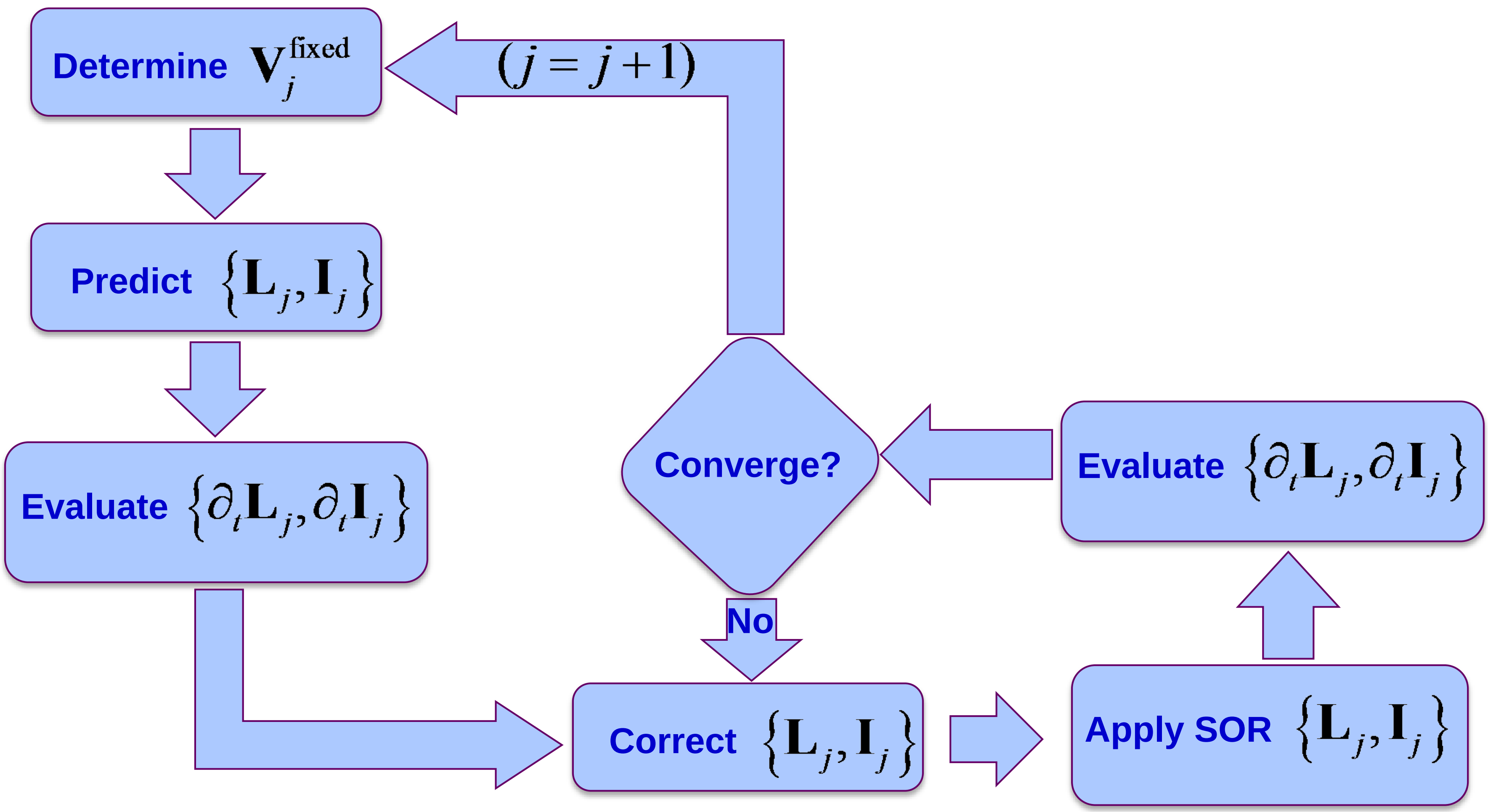
Explicit MOT

- At time step j



Explicit MOT

- At time step j



Numerical Results

- **Excitation is a modulated Gaussian pulse**

$$G(t,f)=\eta_0^{-1}\cos\left[2\pi f(t-t_{\mathrm{p}})\right]e^{-\frac{(t-t_{\mathrm{p}})^2}{2\sigma^2}}$$

  - Center frequency : f
  - Effective bandwidth : f_{bw}
  - Time duration : $\sigma = 3/(2\pi f_{\mathrm{bw}})$
  - Time delay : $t_{\mathrm{p}} = 10\sigma$
  - Intrinsic impedance : $\eta_0 = \sqrt{\mu_0/\varepsilon_0}$

Numerical Results

- **Predictor-corrector coefficients**
  - Highly accurate and stable predictor and corrector scheme[4]
  - Number of steps : $k = 22$
  - Convergence condition : $\delta = 10^{-12}$
- **Temporal interpolation functions** $T(t)$
  - Non-causal band limited interpolation functions[5]
  - Causality retained by complex exponential extrapolation scheme[6]

[4] A. Glaser and V. Rokhlin, *J. of Sci. Comput.*, 38(3), pp. 368-399, 2009.
[5] D. S. Weile, *et al.*, *IEEE Trans. Antennas Propag.*, 52(1), pp. 283–295, 2004.
[6] S. Bin Sayed, *et al.*, *IEEE Trans. Antennas Propag.*,63(7), pp. 3098-3110, 2015.

Numerical Results

- **Isotropic scatterer**

$$\mathbf{H}^{\text{inc}}(\mathbf{r},t) = \hat{\mathbf{y}} G(t - \mathbf{r}\cdot\hat{\mathbf{z}}/c_0, f_0)$$

$$\partial_t \mathbf{M}(\mathbf{r},t) = \partial_t \mathbf{H}(\mathbf{r},t)$$

$f_0 = 150\ \text{MHz}$

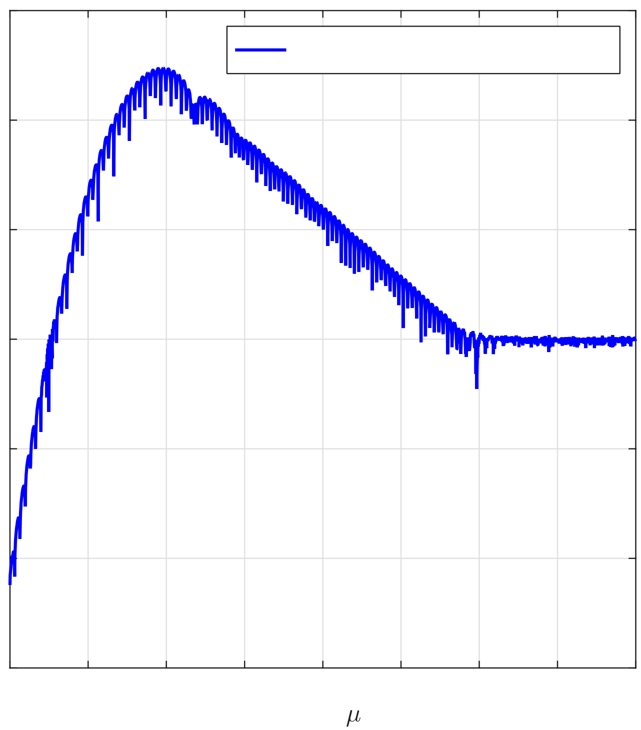
$f_{\text{bw}} = 50\ \text{MHz}$

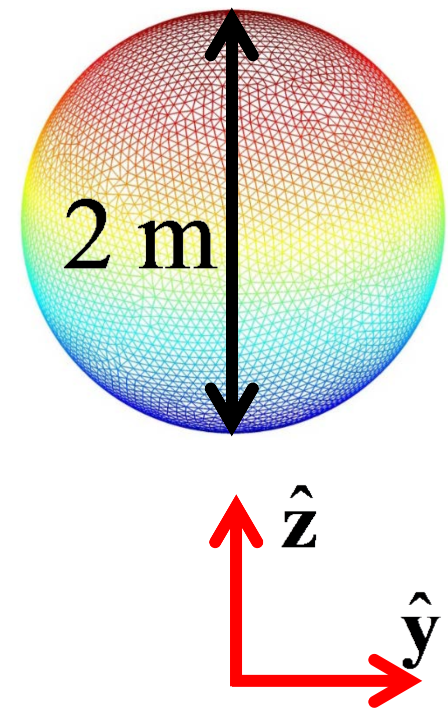
$\Delta t = 0.2\ \text{ns}$

$\mu_{\mathbf{r}} = 2\mu_0$

$N_{\text{v}} = 19692$

$\alpha = 0.6$

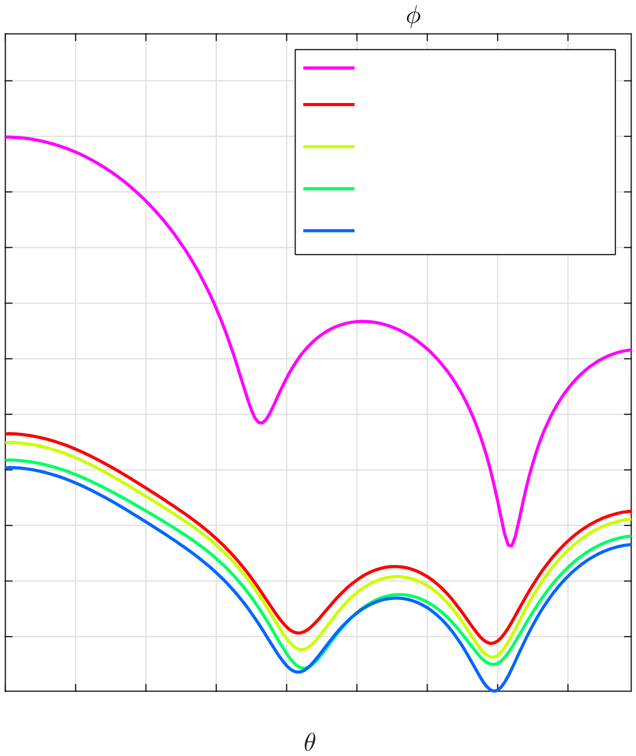


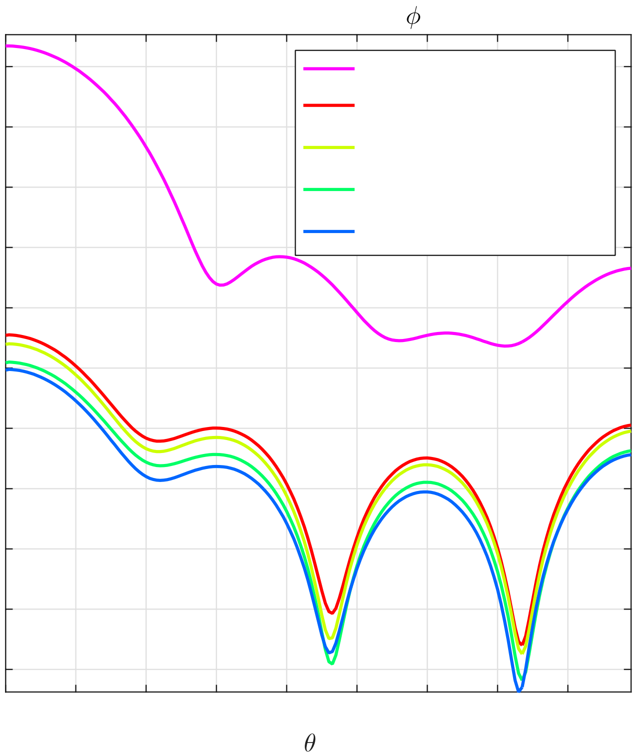


$\mathbf{H}^{\text{inc}}(\mathbf{r},t)$

Numerical Results

- **Isotropic scatterer**





Numerical Results

- **Ferrite scatterer with DC bias**

$$\mathbf{H}^{\mathrm{inc}}(\mathbf{r},t) = \hat{\mathbf{y}}G(t - \mathbf{r}\cdot\hat{\mathbf{z}}/c_0, f_0)$$

$$\mathrm{H}_{\mathrm{dc}} = 2.1420 \text{ Oe} \quad \mathrm{M}_{\mathrm{s}} = 0.7854 \text{ Gauss}$$

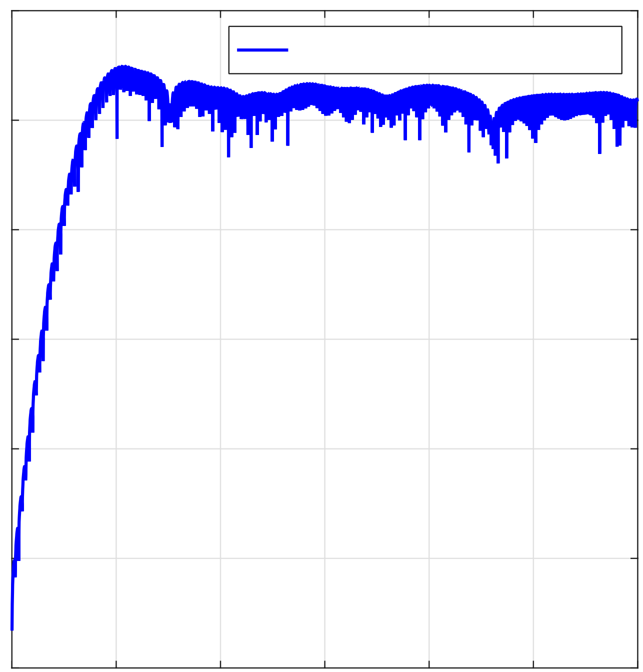
$$f_0 = 150 \text{ MHz}$$

$$f_{\mathrm{bw}} = 50 \text{ MHz}$$

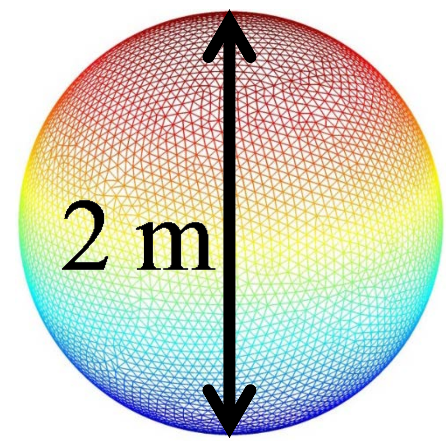
$$\Delta t = 0.2 \text{ ns}$$

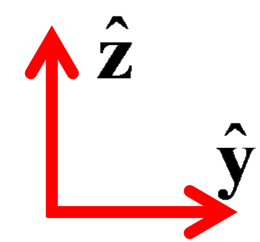
$$N_{\mathrm{v}} = 19692$$

$$\alpha = 0.6$$



μ

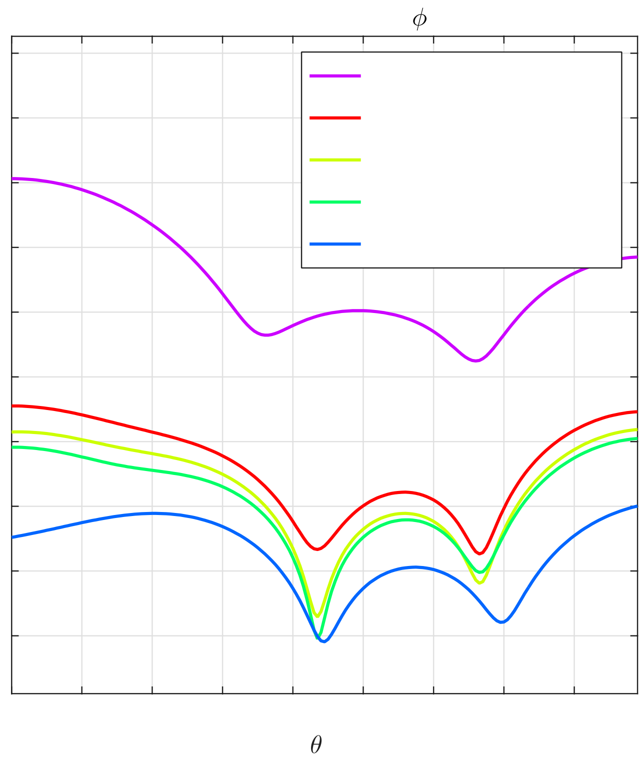


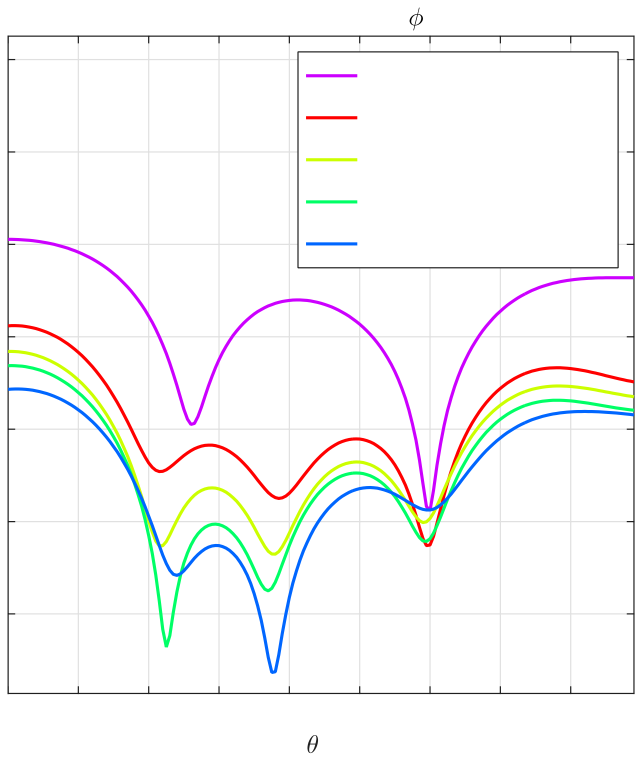


$$\mathbf{H}^{\mathrm{inc}}(\mathbf{r},t)$$

Numerical Results

- **Ferrite scatterer with DC bias**





CEML

Conclusions

- **Summary**
  - **LLG equation is used in the auxiliary form to analyze ferrite scatterers using integral equations**

- **Future work**
  - **Further verification**
  - **Implementation of LLG with no small signal approximation (fully nonlinear)**
  - **Application to “real-life” microwave devices**