A Comparison of Domain Decomposition Techniques for Analysing Disjoint Finite Antenna Arrays

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FEKO
Comprehensive Electromagnetic Solutions

Overview

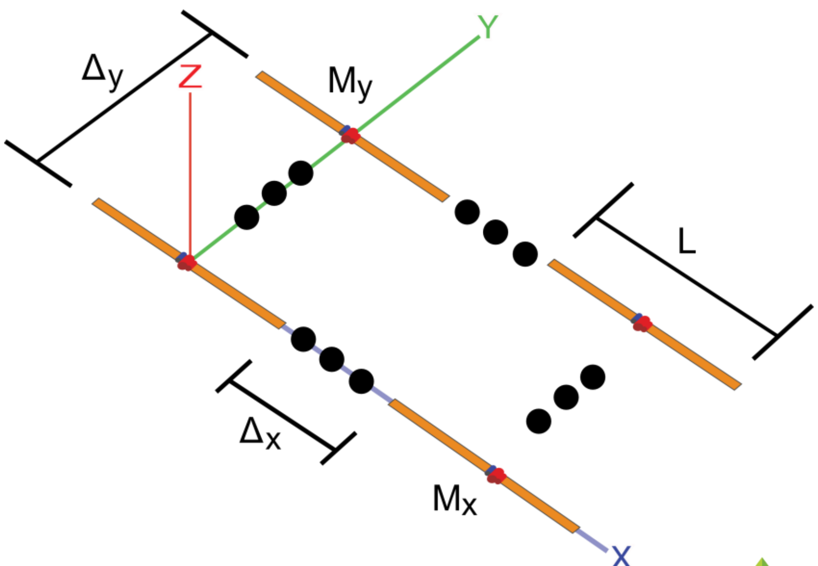
- The Characteristic Basis Function Method (CBFM)
- The Domain Green's Function Method (DGFM)
- The improved DGFM (i-DGFM)
- Computational Complexities (memory and runtime)
- Example – Phased array simulation
- Example – Embedded Element Pattern Calculation
- Conclusions

DOMAIN DECOMPOSITION

Domain Decomposition

- Geometry: Finite Antenna array - M elements, each N_i unknowns each.
- Solving whole problem with MoM (**ZJ=V**) can be quite costly:
  - LU decomposition: $O((M \times N_i)^3)$
  - Memory usage: $O((M \times N_i)^2)$
- Find a way in which to solve the smaller matrix equations for each domain

$$\begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1M} \\ Z_{21} & Z_{22} & \cdots & Z_{2M} \\ \vdots & \vdots & \ddots & \vdots \\ Z_{M1} & Z_{M2} & \cdots & Z_{MM} \end{bmatrix} \begin{bmatrix} J_1 \\ J_2 \\ \vdots \\ J_M \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_M \end{bmatrix}$$



THE CBFM

The CBFM

- Current on *p*th domain is expressed as weighted summation of primary and secondary currents (CBFs):

$$J_p \approx \underbrace{\beta_{0p} J_{0p}}_{\text{Primary CBF}} - \sum_{m=1, m \neq p}^{M} \beta_{pm} \underbrace{Z_{pp}^{-1} Z_{pm} J_{0m}}_{\text{Secondary CBF}}$$

- The primary CBFs are calculated by solving the induced current on domain *p* in isolation:

$$J_{0p} = Z_{pp}^{-1} V_p$$

- By substituting the above expression for **J** into the MoM eq. (**ZJ=V**) we can setup a reduced matrix with each term expressed as a *M x M* sub-matrix as follows:

$$\left[Z_{pq}^{CBFM}\right]_{M \times M} = \left\langle J_p^T, Z_{pq} J_q \right\rangle$$

The CBFM

- The computational complexity for solving the CBFM reduced impedance matrix is much less compared to that of the MoM

	MoM	CBFM
Solving ZJ=V	$O\ ((M \times N_i)^3)$	$O\ ((M^2)^3) = O\ (M^6)$
Memory Usage (Z)	$O\ ((M \times N_i)^2)$	$O\ ((M^2)^2) = O\ (M^4)$

M: Number of array elements. $\boldsymbol{N_i}$ **:** Number of sub sectional basis functions

- Multiple excitations are also treated efficiently – factorisation is done once, followed by a number of backward substitutions for each excitation.

THE DGFM

The DGFM

- A perturbation method (fundamentally based on [1] and [2]) where we make an *initial* assumption that the current on domains *p* and *q* are related:

$$J_q = \alpha_{qp} J_p \quad \text{and} \quad \alpha_{qp} = \frac{C_q}{C_p}$$

***C_q*:** Complex excitation coefficient on domain *p*

- We can now substitute the above into **ZJ=V** to get an active impedance matrix equation for domain *p:*

$$V_p = \left[Z_{pp} + (Z_{pq}\alpha_{qp} + \cdots + Z_{pM}\alpha_{Mp})\right]J_p = \left[\sum_{m=1}^{M} Z_{pm}\alpha_{mp}\right]J_p = Z_p^{act} J_p$$

- Similarly we can solve for the current on domain *q* using:

$$V_q = Z_q^{act} J_q$$

- Note that we have perturbed our initial assumption so that:

$$J_q \neq \alpha_{qp} J_p$$

The DGFM

- We can rewrite our original assumption about the current on domains p and q more generally as:

$$J_q = \alpha_{qp} J_p \quad \text{and} \quad \alpha_{qp} = \frac{\langle \Lambda, J_q \rangle}{\langle \Lambda, J_p \rangle}$$

- Using the terminal voltages to estimate the current ratio on the domains, i.e., $\alpha_{qp} = C_q / C_p$, is identical to the primary CBFs, i.e.,

$$\alpha_{qp} = \frac{\langle \Lambda, J_{0q} \rangle}{\langle \Lambda, J_{0p} \rangle} = \frac{C_q}{C_p}$$

- The DGFM therefore only accounts for 0th-order coupling effects on account of using only the primary CBFs to estimate the currents

The DGFM

- The computational complexity for solving the DGFM active impedance matrices are much less compared to that of the MoM

	MoM	DGFM
Solving ZJ=V	$O\ ((M \times N_i)^3)$	$O\ (M \times N_i^3)$
Memory Usage (Z)	$O\ ((M \times N_i)^2)$	$O\ (N_i)^2$

M: Number of array elements. $\boldsymbol{N_i}$ **:** Number of sub sectional basis functions

- Unlike the CBFM, multiple excitations are not treated as efficiently for the DGFM – the reason being that an active impedance matrix equation is reformulated for each excitation.

THE i-DGFM

The i-DGFM

- We need to make our initial assumption, $J_q = \alpha_{qp} J_p$, as accurate as possible . We do this by expressing the currents on domains *p* and *q* as follows:

$$J_p \approx \underbrace{J_{0p}}_{\text{Primary CBF}} - \sum_{m=1, m \neq p}^{M} \underbrace{Z_{pp}^{-1} Z_{pm} J_{0m}}_{\text{Secondary CBF}}$$

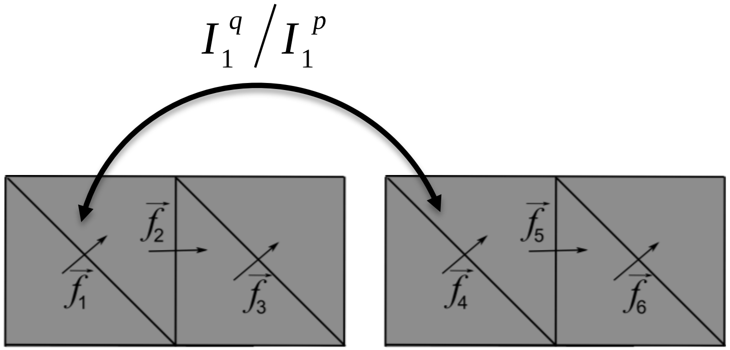
$$J_p \approx \underbrace{J_{0p}}_{\text{Primary CBF}} - \sum_{m=1, m \neq p}^{M} \underbrace{Z_{pp}^{-1} Z_{pm} J_{0m}}_{\text{Secondary CBF}}$$

- Note that we have not included the unknown β coefficients, as done in the CBFM. This is equivalent to taking two Jacobi iterations [3].

The i-DGFM

- Obtain a better expression for the ration of the currents on the domain by using, instead of an α_{qp} constant scaling factor, a diagonal weight matrix as follows:

$$[\alpha_{qp}] = \begin{bmatrix} I_1^q / I_1^p & 0 & 0 & 0 \\ 0 & I_2^q / I_2^p & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & I_N^q / I_N^p \end{bmatrix}$$



- The $[\alpha_{qp}]$ matrix supports more rapid current variation between the domains, as compared to the constant scaling factor

COMPUTATIONAL COMPLEXITIES

Computational Complexity

Method / Phase:	CBFM	DGFM	i-DGFM
MoM matrix setup	$O((M \times N_i)^2)$	$O((M \times N_i)^2)$	$O((M \times N_i)^2)$
Calculating Primary CBFs	$O(M \times N_i^3)$	*NA*	$O(M \times N_i^3)$
Calculating Secondary CBFs	$O((M \times N_i)^2)$	*NA*	$O((M \times N_i)^2)$
Reduced impedance matrix setup	$O(M^4 \times N_i^2)$	*NA*	*NA*
Active impedance matrix setup	*NA*	$O((M \times N_i)^2)$	$O((M \times N_i)^2)$
Solving for $\mathbf{J}_{\mathbf{MoM}}$	$O((M^2)^3) = O(M^6)$	$O(M \times N_i^3)$	$O(M \times N_i^3)$
Memory usage	$O(2N_i^2 + M^4)$	$O(N_i^2)$	$O(N_i^2)$

M: Number of array elements. $\boldsymbol{N_i}$ **:** Number of sub sectional basis functions

Computational Complexity

Method / Phase:	CBFM	DGFM	i-DGFM
MoM matrix setup	$O\ ((M\ x\ N_i)^2)$	$O\ ((M\ x\ N_i)^2)$	$O\ ((M\ x\ N_i)^2)$
	$O\ (M\ x\ N_i^3)$	*NA*	$O\ (M\ x\ N_i^3)$
	$O\ ((M\ x\ N_i)^2)$	*NA*	$O\ ((M\ x\ N_i)^2)$
	$O\ (M^4\ x\ N_i^2)$	*NA*	*NA*
	NA	$O\ ((M\ x\ N_i)^2)$	$O\ ((M\ x\ N_i)^2)$
	$O\ ((M^2)^3) = O\ (M^6)$	$O\ (M\ x\ N_i^3)$	$O\ (M\ x\ N_i^3)$
	$O\ (2N_i^2 + M^4)$	$O\ (N_i^2)$	$O\ (N_i^2)$

M: Number of array elements. $\boldsymbol{N_i}$ **:** Number of sub sectional basis functions

Computational Complexity

Method / Phase:	CBFM	DGFM	i-DGFM
	$O((M \times N_i)^2)$	$O((M \times N_i)^2)$	$O((M \times N_i)^2)$
Calculating Primary CBFs	$O(M \times N_i^3)$	*NA*	$O(M \times N_i^3)$
Calculating Secondary CBFs	$O((M \times N_i)^2)$	*NA*	$O((M \times N_i)^2)$
	$O(M^4 \times N_i^2)$	*NA*	*NA*
	NA	$O((M \times N_i)^2)$	$O((M \times N_i)^2)$
	$O((M^2)^3) = O(M^6)$	$O(M \times N_i^3)$	$O(M \times N_i^3)$
	$O(2N_i^2 + M^4)$	$O(N_i^2)$	$O(N_i^2)$

M: Number of array elements. $\boldsymbol{N_i}$ **:** Number of sub sectional basis functions

Computational Complexity

Method / Phase:	CBFM	DGFM	i-DGFM
	$O\ ((M\ x\ N_i)^2)$	$O\ ((M\ x\ N_i)^2)$	$O\ ((M\ x\ N_i)^2)$
	$O\ (M\ x\ N_i^3)$	*NA*	$O\ (M\ x\ N_i^3)$
	$O\ ((M\ x\ N_i)^2)$	*NA*	$O\ ((M\ x\ N_i)^2)$
Reduced impedance matrix setup	$O\ (M^4\ x\ N_i^2)$	*NA*	*NA*
Active impedance matrix setup	*NA*	$O\ ((M\ x\ N_i)^2)$	$O\ ((M\ x\ N_i)^2)$
	$O\ ((M^2)^3) = O\ (M^6)$	$O\ (M\ x\ N_i^3)$	$O\ (M\ x\ N_i^3)$
	$O\ (2N_i^2 + M^4)$	$O\ (N_i^2)$	$O\ (N_i^2)$

M: Number of array elements. $\boldsymbol{N_i}$ **:** Number of sub sectional basis functions

Computational Complexity

Method / Phase:	CBFM	DGFM	i-DGFM
	$O((M \times N_i)^2)$	$O((M \times N_i)^2)$	$O((M \times N_i)^2)$
	$O(M \times N_i^3)$	NA	$O(M \times N_i^3)$
	$O((M \times N_i)^2)$	NA	$O((M \times N_i)^2)$
	$O(M^4 \times N_i^2)$	NA	NA
	NA	$O((M \times N_i)^2)$	$O((M \times N_i)^2)$
Solving for $\mathbf{J}_{\mathbf{MoM}}$	$O((M^2)^3) = O(M^6)$	$O(M \times N_i^3)$	$O(M \times N_i^3)$
	$O(2N_i^2 + M^4)$	$O(N_i^2)$	$O(N_i^2)$

M: Number of array elements. $\boldsymbol{N_i}$ **:** Number of sub sectional basis functions

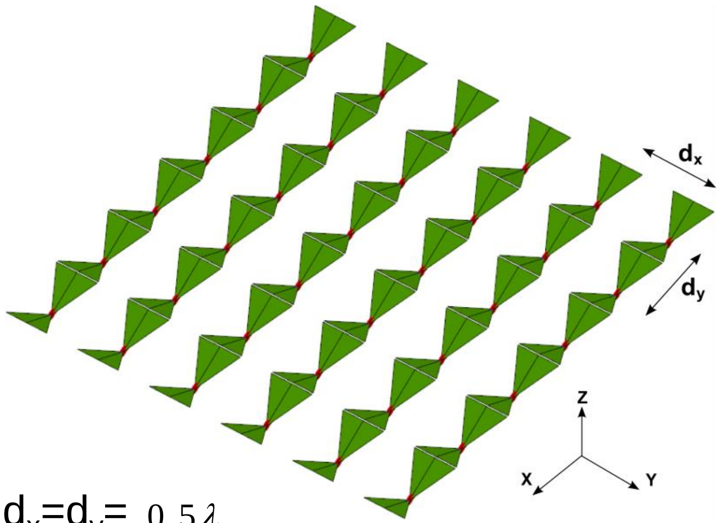
Computational Complexity

Method / Phase:	CBFM	DGFM	i-DGFM
	$O((M \times N_i)^2)$	$O((M \times N_i)^2)$	$O((M \times N_i)^2)$
	$O(M \times N_i^3)$	NA	$O(M \times N_i^3)$
	$O((M \times N_i)^2)$	NA	$O((M \times N_i)^2)$
	$O(M^4 \times N_i^2)$	NA	NA
	NA	$O((M \times N_i)^2)$	$O((M \times N_i)^2)$
	$O((M^2)^3) = O(M^6)$	$O(M \times N_i^3)$	$O(M \times N_i^3)$
Memory usage	$O(2N_i^2 + M^4)$	$O(N_i^2)$	$O(2N_i^2)$

M: Number of array elements. $\boldsymbol{N_i}$ **:** Number of sub sectional basis functions

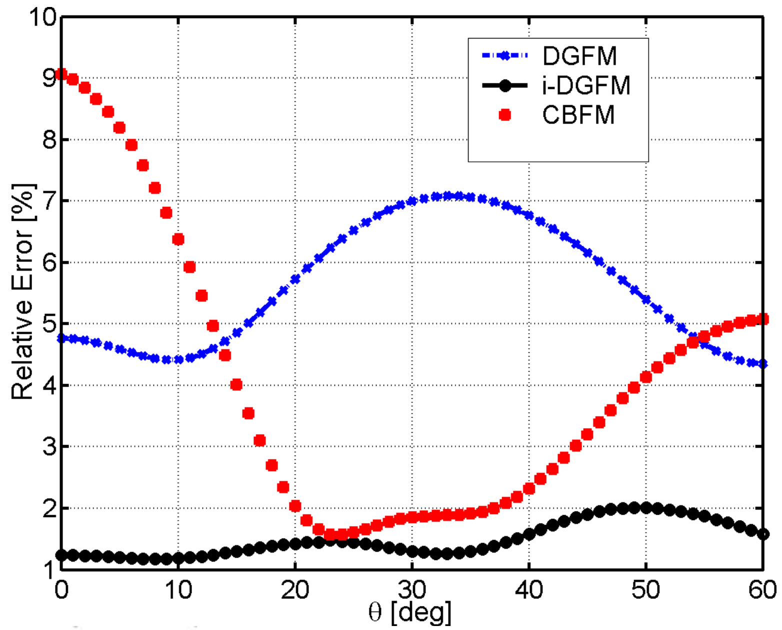
EXAMPLE – PHASED ARRAY SIMULATION

Example – Phased Array simulation



d_x=d_y= 0.5λ
N_{tot} = 2,844 RWGs
Prim. CBFs/domain: 1
Sec. CBFs/domain: 35

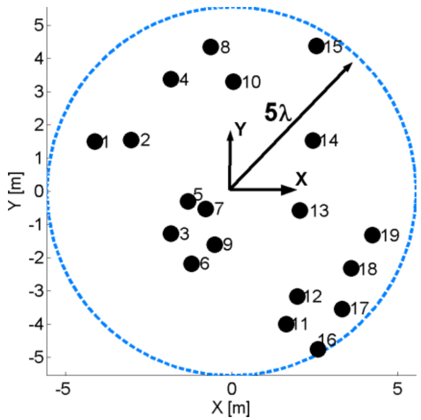
$$\varepsilon_{\%} = \sqrt{\sum_{n}^{N^{RWG}} \left| I_n^{ref} - I_n \right|^2 \Big/ \sum_{n}^{N^{RWG}} \left| I_n^{ref} \right|^2} \times 100\ \%$$



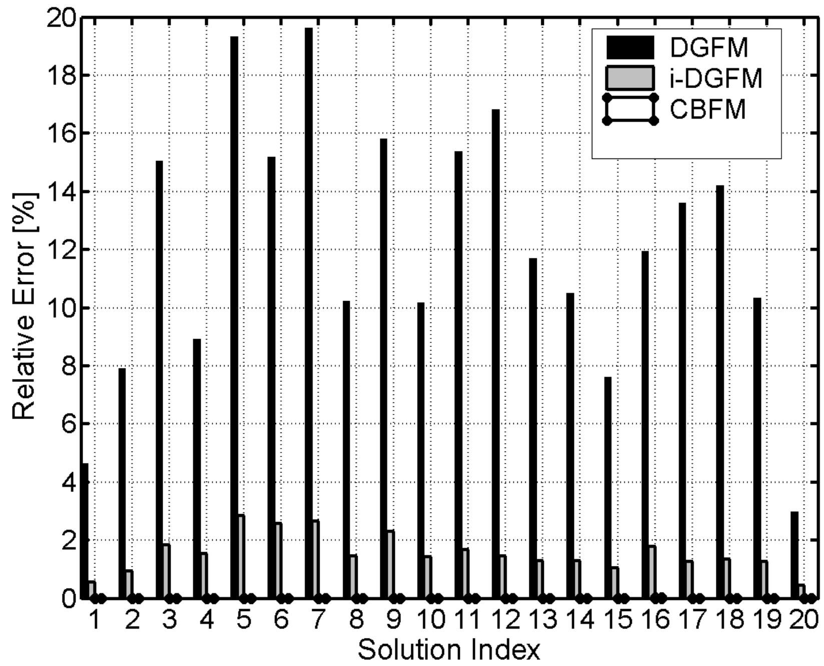
Method	Solution time	Memory usage
MoM	33 sec	123.42 MByte
DGFM	0.703 sec	97.52 kByte
i-DGFM	4.281 sec	195.4 kByte
CBFM	7.187 sec	25.63 MByte
Calculating prim. CBFs	1.122 sec	44.44 kByte
Calculating second. CBFs	74.654 sec	1.52 MByte

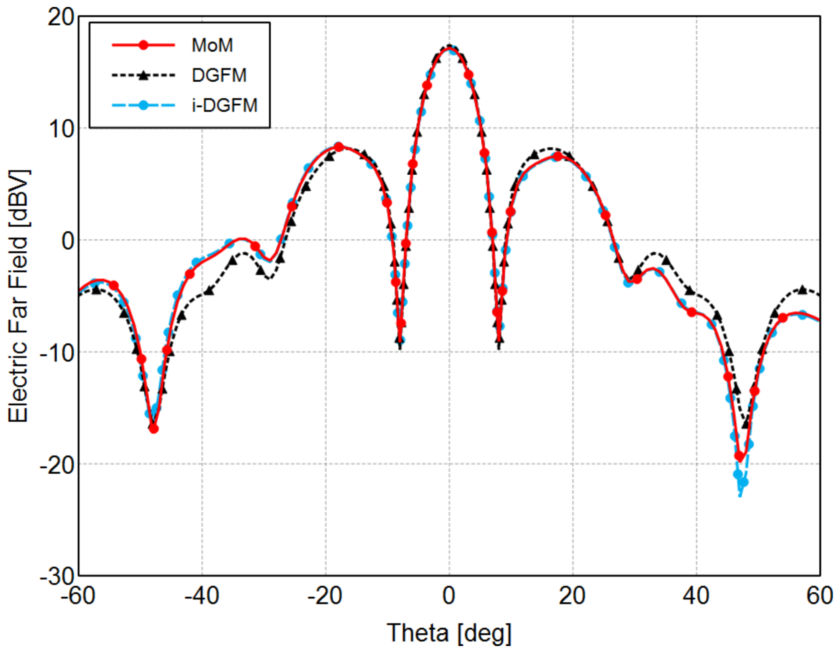
EXAMPLE – EMBEDDED ELEMENT PATTERN CALCULATION

Example – Phased Array simulation



N_{tot} = 2,109 RWGs
Prim. CBFs/domain: 1
Sec. CBFs/domain: 18





Method	Solution time	Memory usage
MoM	18.64 sec	67.87 MByte
DGFM	0.953 sec	192.52 kByte
i-DGFM	3.734 sec	385.04 kByte
CBFM	23.156 sec	1.99 MByte
Calculating prim. CBFs	0.03 sec	32.95 kByte
Calculating second. CBFs	1.236 sec	593.16 kByte

Conclusions

Method / Phase:	CBFM	DGFM	i-DGFM
Accuracy (Active Arrays – tightly coupled)	✔	~	✔
Accuracy (Passive Arrays)	✔	✗	~
Multiple excitations	✔	~	~
Runtime complexity	$O\ (MN_i^3 + M^2N_i^3 + M^4N_i^2 + M^6)$	$O\ (M^2N_i^2 + MN_i^3)$ ✔	$O\ (MN_i^3 + M^2N_i^3 + M^2N_i^2 + MN_i^3)$
Memory usage	$O\ (2N_i^2 + M^4)$	$O\ (N_i^2)$	$O\ (2N_i^2)$
Integration into existing MoM codes	~	✔	~

References

DGFM

[1] A. K. Skriverik and J. R. Mosig, “Analysis of finite phase arays of microstrip patches,” IEEE Transactions on Antennas and Propagation, vol 41, no. 8, pp. 1105–1114, 1993.

[2] D. J. Ludick, R. Maaskant, D. B. Davidson, U. Jakobus, R. Mittra, and D. de Villiers, “Efficient Analysis of Large Aperiodic Antenna Arrays using the Domain Greens Function Method,” IEEE Transactions on Antennas and Propagation, vol. 62, no. 4, 2014.

Jacobi Iterations

[3] Y. Brand, A. K. Skrivervik, J. R. Mosig, and F. E. Gardiol, “New iterative integral equation technique for multilayered printed array antennas,” in Mathematical Methods in Electromagnetic Theory, Kharkov, Ukraine, Jun. 1998, pp. 615–617.

CBFM-enhanced DGFM

[4] D. J. Ludick, R. Maaskant, R. Mittra, , U. Jakobus, and D. B. Davidson, “Applying the CBFM-Enhanced DGFM to the Analysis of Large Finite Antenna Arrays,” in 2013 International Conference on Electromagnetics in Advanced Applications (ICEAA), 2013.