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2016 INTERNATIONAL SYMPOSIUM ON
ANTENNAS AND PROPAGATION
October 24-28, 2016, Okinawa, JAPAN

Relations between the Characteristic Modes(CMs) and the X Modes(XMs)

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Abstract

The differences between the characteristic modes (CMs) and X modes (XMs) are pointed out. Then the relations between these two kinds of modes are discussed through two examples, namely, a thin strip dipole and a rectangular loop. It is found that CMs and XMs have similar resonant frequencies (RFs) and resonant currents (RCs). Finally, it is concluded that the resonant information of an antenna, which includes RFs and RCs, exists only in the imaginary part of the impedance operator (namely, reactance operator) of the antenna.

Index Terms: Characteristic Modes (CMs), X Modes (XMs), relations, impedance operator, resonant frequencies (RFs), resonant currents (RCs), antenna theory.

Biography

<u>Jiang-Feng Lin</u> (S'16) was born in Shantou, Guangdong Province, China, in 1991. He received the B.S. in communication engineering from Beijing University of Posts and Telecommunications, Beijing, China, in 2013. He is currently pursuing the Ph.D. degree at South China University of Technology, Guangzhou, China. His research interests include characteristic modes theory and wideband antennas.

Qing-Xin Chu (M'99–SM'11–F'18) received the B.S., M.E., and Ph.D. degree in electronic engineering from Xidian University, Xi'an, Shaanxi, China, in 1982, 1987, and 1994, respectively.

He is currently a chair professor with the School of Electronic and Information Engineering, South China University of Technology. He is also the director of the Research Institute of Antennas and RF Techniques of the university, the chair of the Engineering Center of Antennas and RF Techniques of Guangdong Province. He is also with Xidian University as a distinguished professor in Shaanxi Hundred-Talent Program since 2011. From Jan. 1982 until Jan. 2004, he was with the School of Electronic Engineering, Xidian University, and since 1997, he was a professor and the vice dean of the School of Electronic Engineering, Xidian University.

He is the foundation chair of IEEE Guangzhou AP/MTT Chapter, fellow of IEEE and the China Electronic Institute (CEI). He has published over 300 papers in journals and conferences, which were indexed in SCI more than 1500 times. One of his papers published in IEEE Transactions on Antennas and Propagations in 2008 becomes the top ESI (Essential Science Indicators) paper within 10 years in the field of antenna (SCI indexed self-excluded in the antenna field ranged top 1%). In 2014, he was elected as the highly cited scholar by Elsevier in the field of Electrical and Electronic Engineering. He has authorized more than 30 invention patents of China.

He was the recipient of the Science Award by Guangdong Province in 2013，the Science Awards by the Education Ministry of China in 2008 and 2002, the Fellowship Award by Japan Society for Promotion of Science (JSPS) in 2004, the Singapore Tan Chin Tuan Exchange Fellowship Award in 2003, the Educational Award by Shaanxi Province in 2003.

His current research interests include antennas in wireless communication, microwave filters, spatial power combining array, and numerical techniques in electromagnetics.

Outline

- Research motivation
- CMs and XMs and their relations
- Examples: a dipole and a rectangular loop
- Conclusions

I. Research Motivation

- Since CMs are extracted from Z operator, all of the information of CMs must be contained in the Z operator. As the real and imaginary parts of Z operator, it's reasonable to conceive that the R operator and the X operator should have different effects on CMs.

- Besides, considering that CMs are antennas' intrinsic properties, it would be helpful for us to understand the radiation mechanism of antennas more deeply once we find how the CMs come into being under the different effects of R and X operators.

- **However, until we write this paper, it seems that the different effects of R and X operators on CMs have not been systematically studied.**

- **Our goal is to make the different effects clear.**

- **Today we will focus on the effects of X operator and the effects of R operator would be further studied in future .**

II. CMs and XMs and their relations

CMs are defined by the following generalized eigenvalue equation:

$$XJ_n^{CM} = \lambda_n^{CM} R J_n^{CM}$$

where λ_n^{CM} and J_n^{CM} represent the characteristic value (CV) and characteristic current (CC) of the n_{th} CM, R and X are the resistance and reactance operators, namely

$$R = (Z + Z^*)/2, X = (Z - Z^*)/2j$$

Here, the asterisk denotes the complex conjugate. Obviously, both R and X are real symmetric operators since Z is a symmetric operator.

To ensure the uniqueness, CMs are normalized as

$$\left\langle J_m^{CM}, RJ_n^{CM} \right\rangle = \delta_{mn}$$

$$\left\langle J_m^{CM}, XJ_n^{CM} \right\rangle = \delta_{mn} \lambda_n^{CM}$$

With the normalization, the characteristic fields (CFs) of CMs are orthogonal with each other as [2]

$$\frac{1}{\eta} \iint_{S'} \boldsymbol{E}_m \cdot \boldsymbol{E}_n^* ds = \delta_{mn}$$

$$\eta \iint_{S'} \boldsymbol{H}_m \cdot \boldsymbol{H}_n^* ds = \delta_{mn}$$

II. CMs and XMs and their relations

The characteristic values (CVs) are closely associated with the following Rayleigh quotient:

$$\lambda_n^{CM} = \frac{\langle J_n, XJ_n \rangle}{\langle J_n, RJ_n \rangle}$$

The complex power P_{in} can be computed as

$$P_{in} = P_{rad} + 2j\omega(W_m - W_e)$$

$$P_{rad} = \langle J, RJ \rangle$$

$$2\omega(W_m - W_e) = \langle J, XJ \rangle$$

where W_e and W_m denote the time averaged stored electric and magnetic energies and P_{rad} denotes the radiated power.

It can be easily derived that J_n^{CM} radiates a constant power of 1w and the time average energy it stores is $\lambda_n^{CM} / 2\omega$ by combining all the equations above.

If $\lambda_n^{CM}(f_{res}) = 0$, we call f_{res} as the resonant frequency (RF) and $J_n^{CM}(f_{res})$ as the resonant current (RC) of the n_{th} CM. Then we have

$$\langle J_n(f_{res}), XJ_n(f_{res}) \rangle = 0$$

From the left part of the above equation we can see RF and RC are only associated with X operator and have nothing to do with R operator. In other words, X operator determines the RFs and RCs of CMs.
To verify this observation, a new kind of X modes (XMs) are proposed as:

$$XJ_n^{XM} = \lambda_n^{XM} J_n^{XM}$$

Similar to CMs, we call f_{res} as the RF and $J_n^{XM}(f_{res})$ as the RC of the n_{th} XM if $\lambda_n^{XM}(f_{res}) = 0$.

II. CMs and XMs and their relations

$$XJ_n^{CM} = \lambda_n^{CM} RJ_n^{CM}$$
$$\langle J_m^{CM}, RJ_n^{CM} \rangle = \delta_{mn}$$
$$\langle J_m^{CM}, XJ_n^{CM} \rangle = \delta_{mn}\lambda_n^{CM}$$

Sharing the same RFs and RCs

$$XJ_n^{XM} = \lambda_n^{XM} J_n^{XM}$$
$$\langle J_m^{XM}, J_n^{XM} \rangle = \delta_{mn}$$
$$\langle J_m^{XM}, XJ_n^{XM} \rangle = \delta_{mn}\lambda_n^{XM}$$

$$P_{in} = P_{rad} + 2j\omega(W_m - W_e)$$
$$P_{rad} = \langle J_n, RJ_n \rangle$$
$$2\omega(W_m - W_e) = \langle J_n, XJ_n \rangle$$

$$\lambda_n^{CM} = \frac{\langle J_n^{CM}, XJ_n^{CM} \rangle}{\langle J_n^{CM}, RJ_n^{CM} \rangle} \qquad \lambda_n^{XM} = \frac{\langle J_n^{XM}, XJ_n^{XM} \rangle}{\langle J_n^{XM}, J_n^{XM} \rangle}$$

$$\langle J_n(f_{res}), XJ_n(f_{res}) \rangle = 0$$

Their CFs are different

CFs of CMs are orthogonal

$$<J_m, RJ_n> = \delta_{mn}$$
$$<J_m, XJ_n> = \lambda_n\delta_{mn}$$
$$\Rightarrow <J_m, ZJ_n> = (1 + j\lambda_n)\delta_{mn}$$
$$\Rightarrow \frac{1}{\eta}\iint_{S'} E_m \cdot E_n^* ds = \delta_{mn}, \quad \eta\iint_{S'} H_m \cdot H_n^* ds = \delta_{mn}$$

CFs of XMs are not orthogonal

$$<J_m, J_n> = \delta_{mn}$$
$$<J_m, XJ_n> = \lambda_n\delta_{mn}$$
$$\nRightarrow <J_m, ZJ_n> = (1 + j\lambda_n)\delta_{mn}$$

II. CMs and XMs and their relations

To some extent, CMs can be seen as the XMs which have been further modulated by the R operator. The modulation effect enables the CFs of XMs orthogonal with each other.

$$\left\langle J_m^{CM}, RJ_n^{CM} \right\rangle = \delta_{mn}$$
$$\left\langle J_m^{CM}, XJ_n^{CM} \right\rangle = \delta_{mn}\lambda_n^{CM}$$

$$\left\langle J_m^{XM}, J_n^{XM} \right\rangle = \delta_{mn}$$
$$\left\langle J_m^{XM}, XJ_n^{XM} \right\rangle = \delta_{mn}\lambda_n^{XM}$$

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III. Examples

In this section, we would verify that CMs and XMs have identical RFs and RCs.

But before this, we have to calculate the CMs and XMs at first. We use the RWG functions {Wi} (which are always used in standard Moment Method) to convert the operator equations on the left into matrix equations on the right:

$$XJ_n^{CM} = \lambda_n^{CM} R J_n^{CM}$$

$$XJ_n^{XM} = \lambda_n^{XM} J_n^{XM}$$

RWG functions →

$$[R] = \left[\langle w_i, Rw_j \rangle\right]$$

$$[X] = \left[\langle w_i, Xw_j \rangle\right]$$

$$[X]\left[I_n^{CM}\right] = \lambda_n^{CM}[R]\left[I_n^{CM}\right]$$

$$[X]\left[I_n^{XM}\right] = \lambda_n^{XM}\left[I_n^{XM}\right]$$

III. Examples: a thin strip dipole

With a length of 100mm and a width of 1mm , the dipole is simulated from 1 to 6.5 GHz in 10 MHz steps. The largest dimension of the triangular meshes is 1mm.

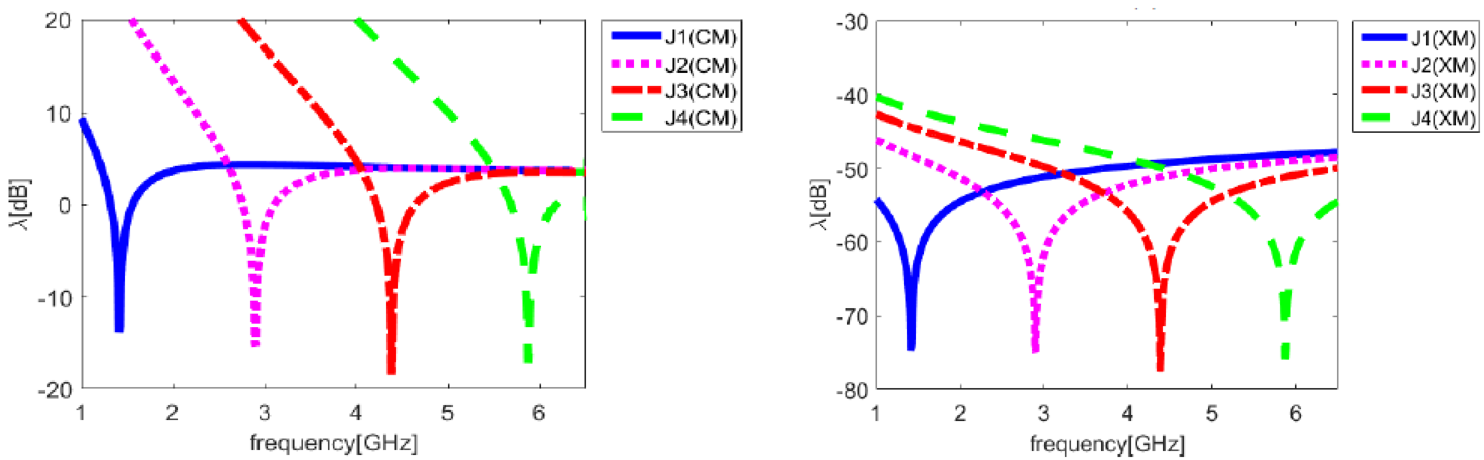
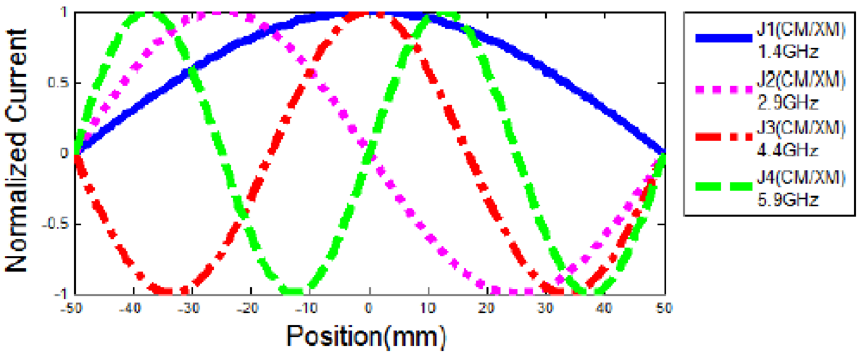


Fig.1 shows the variation with frequency of the eigenvalues of the first four CMs and XMs. It can be found that CMs and XMs have the same modal resonant frequencies which locate at 1.4GHz, 2.9 GHz, 4.4 GHz and 5.9 GHz, respectively. In other words, CMs and XMs share the same resonant frequency information of the dipole.

III. Examples: a thin strip dipole

In spite of their different magnitudes, the normalized resonant current distributions of CMs and XMs are found to be the same intuitively, as shown in Fig.2.



To further study how CMs resemble XMs in a quantitative way, it seems to be beneficial to use the correlation coefficient (CC) which is defined as

$$corr(I_n^{CM}, I_n^{XM}) = \frac{\left|\sum_i (I_{n,i}^{CM} - \bar{I}_n^{CM})(I_{n,i}^{XM} - \bar{I}_n^{XM})\right|}{\sqrt{\sum_i (I_{n,i}^{CM} - \bar{I}_n^{CM})^2 \sum_j (I_{n,j}^{XM} - \bar{I}_n^{XM})^2}}$$

$$0 \le corr(I_n^{CM}, I_n^{XM}) \le 1$$

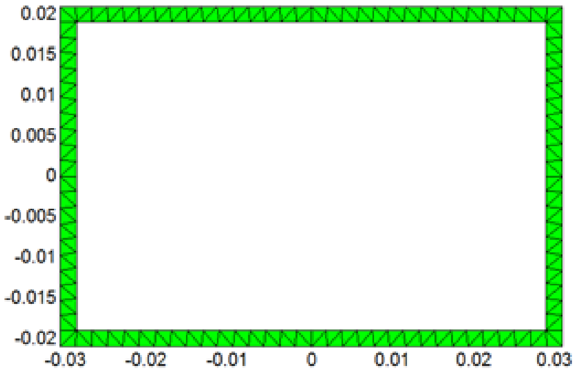
TABLE I
Correlation Coefficients of CMs and XMs

CM,XM	I_1^{CM}, I_1^{XM}	I_2^{CM}, I_2^{XM}	I_3^{CM}, I_3^{XM}	I_4^{CM}, I_4^{XM}
FRE	1.4GHz	2.9GHz	4.4GHz	5.9GHz
CORR	1	1	1	1

It can be seen that the CC is normalized implicitly and it does not depend on the magnitude of the resonant current for CMs and XMs.
Then the CCs between CMs and XMs are computed and they are found all equal to 1 as shown in Table I. This fact means that the resonant currents of CMs and XMs are really the same.
So for a dipole, its CMs and XMs sharing the same RFs and RCs (despite the magnitudes of RCs).

III. Examples: a rectangular loop

As shown in Fig. 3, the rectangular loop has an inner length of 58mm , an outer length of 62mm , an inner width of 38mm, and an outer width of 42mm . Both the width of the sides and the largest dimension of the triangular meshes are 2mm. The simulated frequency band covers from 0.5 to 6.5 GHz with a step of 10MHz.



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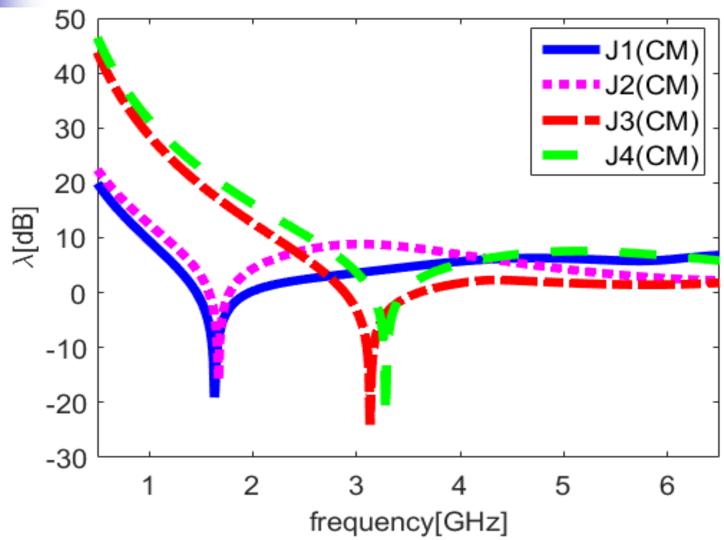
III. Examples: a rectangular loop



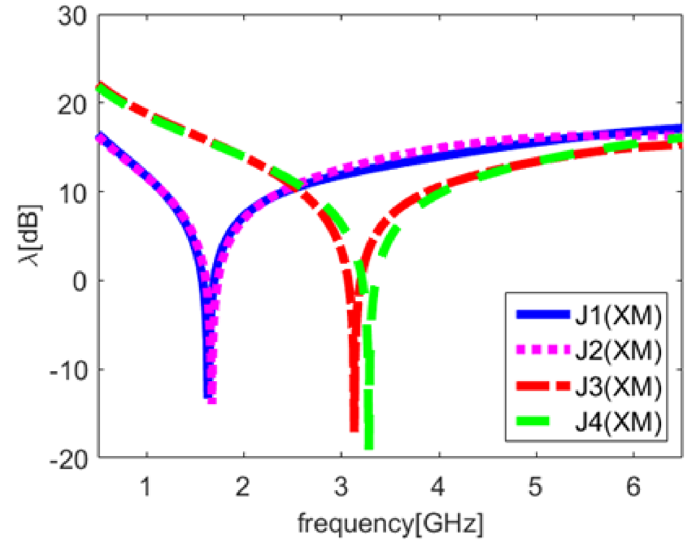
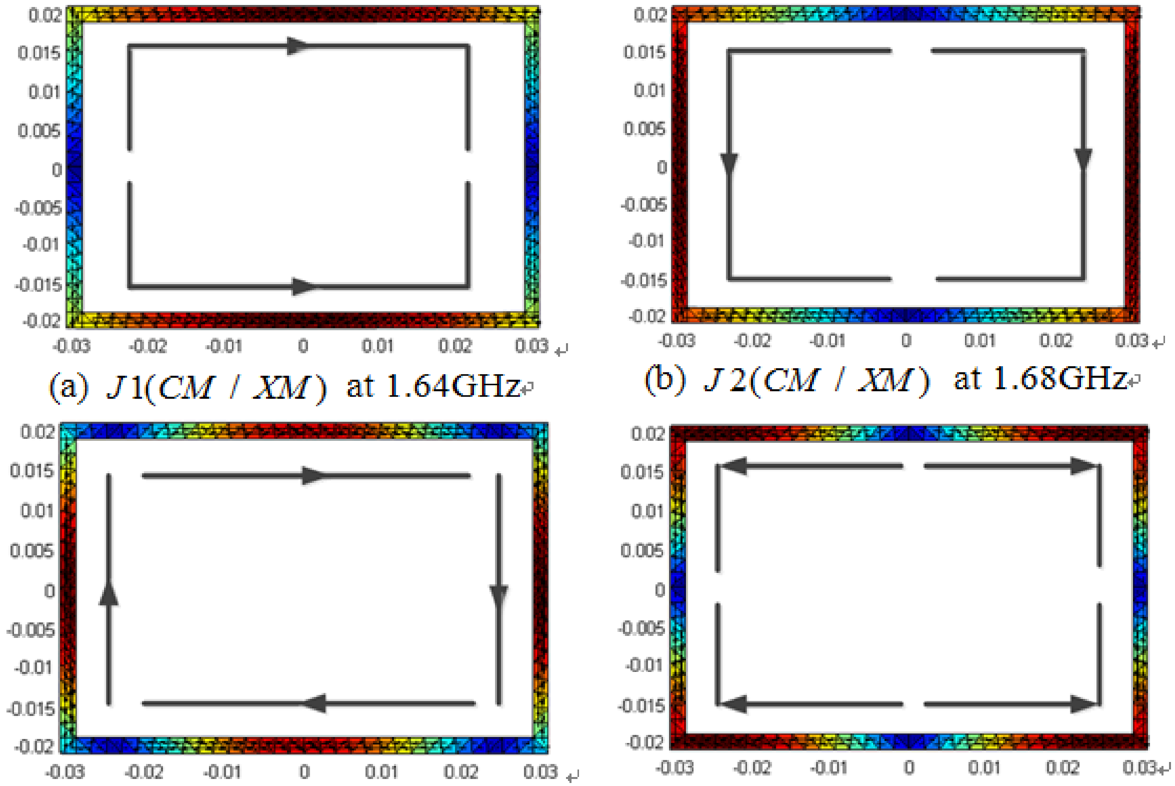


Fig. 4 shows the variation with frequency of the CVs of the first four CMs and XMs. Similar with the dipole case, CMs and XMs of the rectangular loop have the same RFs.

III. Examples: a rectangular loop



(a) $J1(CM\ /\ XM)$ at 1.64GHz

(b) $J2(CM\ /\ XM)$ at 1.68GHz

(c) $J3(CM\ /\ XM)$ at 3.14GHz

(d) $J4(CM\ /\ XM)$ at 3.29GHz

Fig. 5 shows the normalized RCs of CMs and XMs. As we can see, the RCs of CMs and XMs are the same in spite of their different magnitudes.

VI. Conclusion

- By comparing XMs with the conventional CMs, it has been found that it's the X operator containing the resonant information (including RFs and RCs) of CMs.
- Considering that CMs are the intrinsic property of antennas, we can further conclude that X operator contains the resonant information of the antenna.
- We have found the effects of the X operator on CMs but leaving the R operator to be further studied in future.

References

- Garbacz, R.J. and Turpin, R., "A generalized expansion for radiated and scattered fields," in *IEEE Transactions on Antennas and Propagation*, vol.19, no.3, pp.348-358, May 1971.
- Harrington, Roger F. and Mautz, J.R., "Theory of characteristic modes for conducting bodies," in *IEEE Transactions on Antennas and Propagation*, vol.19, no.5, pp.622-628, Sep 1971.
- Harrington, Roger F. and Mautz, J.R., "Computation of characteristic modes for conducting bodies," in *IEEE Transactions on Antennas and Propagation*, vol.19, no.5, pp.629-639, Sep 1971.
- M. Cabedo-Fabres, E. Antonino-Daviu, A. Valero-Nogueira and M. F. Bataller, "The Theory of Characteristic Modes Revisited: A Contribution to the Design of Antennas for Modern Applications," in *IEEE Antennas and Propagation Magazine*, vol. 49, no. 5, pp. 52-68, Oct. 2007.
- K. R. Schab and J. T. Bernhard, "Radiation and Energy Storage Current Modes on Conducting Structures," in *IEEE Transactions on Antennas and Propagation*, vol. 63, no. 12, pp. 5601-5611, Dec. 2015.
- S. Rao, D. Wilton and A. Glisson, "Electromagnetic scattering by surfaces of arbitrary shape," in *IEEE Transactions on Antennas and Propagation*, vol. 30, no. 3, pp. 409-418, May 1982.

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